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TABLE OF SPECTRA.

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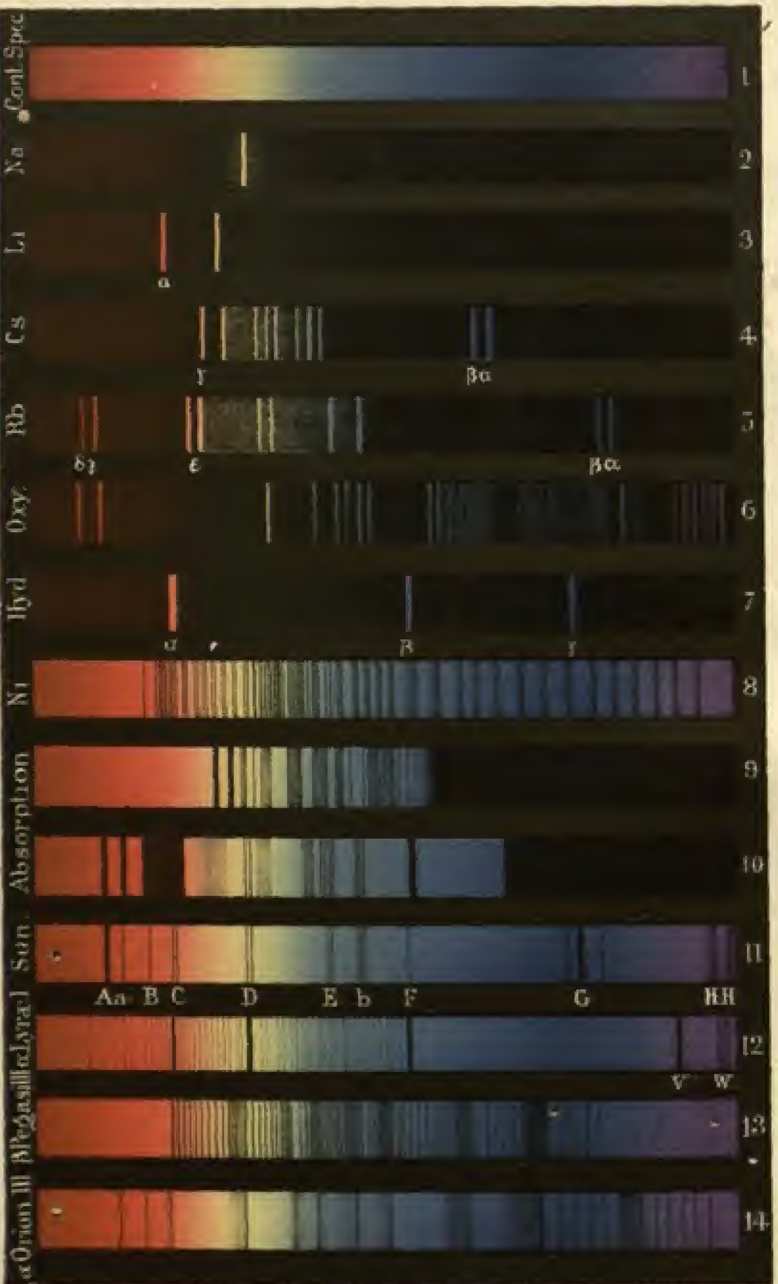

ELEMENTARY TREATISE

ON

PHYSICS

EXPERIMENTAL AND APPLIED

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ELEMENTARY TREATISE

ON

PHYSICS

EXPERIMENTAL AND APPLIED

FOR THE USE OF COLLEGES AND SCHOOLS.

TRANSLATED AND EDITED FROM

GANOT'S ÉLÉMENTS DE PHYSIQUE

(with the Author's sanction)

BY

E. ATKINSON, PH.D., F.C.S.

PROFESSOR OF EXPERIMENTAL SCIENCE, STAFF COLLEGE, SANDHURST.

Tenth Edition, revised and enlarged.

ILLUSTRATED by 4 COLOURED PLATES and 844 WOODCUTS.

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TO

THE TENTH EDITION.

IN THE PRESENT EDITION the fresh matter has increased by about twenty-five pages the size of the book as it stood in the last Edition. The new matter includes twenty-four additional illustrations.

The continued and even increasing favour with which the work has been received, both as a Text Book for Colleges and Schools, and also as a work of reference for the general reader, renders any apology for omissions perhaps unnecessary; it may, however, be as well once more to point out that the book is intended to be a general Elementary Treatise on Physics; and that, while it accordingly aims at giving an account of the most important facts and general laws of all branches of Physics, an attempt to treat completely and exhaustively of any one branch, would both be inconsistent with the general plan of the book, and impossible within the available space.

E. A.

STAFF COLLEGE: *April* 1881.

EXTRACT FROM ADVERTISEMENT TO THE SEVENTH EDITION.

I HAVE ADDED an Appendix containing a series of numerical problems and examples in Physics. This Appendix is based upon a similar one contained in the French edition of the work. But I have been able to use only a small proportion of the problems contained in that Appendix, as the interest of the solution was in most cases geometrical or algebraical. Hence I have substituted or added others, which have been so selected as to involve in the solution a knowledge of some definite physical principle.

Such an Appendix has from time to time been urged upon me by teachers and others who use the work. It will, I conceive, be most useful to those students who have not the advantage of regular instruction; affording to them a means of personally testing their knowledge. Such a student should not aim solely at getting a result which numerically agrees with the answer. He should habituate himself to write out at length the several steps by which the result is obtained, so that he may bring clearly before himself the physical principles involved in each stage. Some of the solutions of the problems are therefore worked out at length.

E. A.

TRANSLATOR'S PREFACE to FIRST EDITION.

THE *Éléments de Physique* of Professor GANOT, of which the present work is a translation, has acquired a high reputation as an Introduction to Physical Science. In France it has passed through Nine large editions in little more than as many years, and it has been translated into German and Spanish.

This reputation it doubtless owes to the clearness and conciseness with which the principal physical laws and phenomena are explained, to its methodical arrangement, and to the excellence of its illustrations. In undertaking a translation, I was influenced by the favourable opinion which a previous use of it in teaching had enabled me to form.

I found that its principal defect consisted in its too close adaptation to the French systems of instruction; and accordingly, my chief labour, beyond that of mere translation, has been expended in making such alterations and additions as might render it more useful to the English student.

I have retained throughout the use of the Centigrade thermometer, and in some cases have expressed the smaller linear measures on the metrical system. These systems are now everywhere gaining ground, and an apology is scarcely needed for an innovation which may help to familiarise the English student with their use in the perusal of the larger and more complete works on Physical Science to which this work may serve as an introduction.

E. ATKINSON.

ROYAL MILITARY COLLEGE, SANDHURST,
1863.

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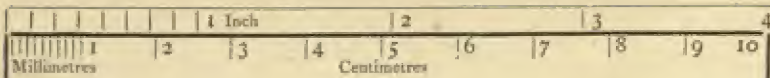

The area of the figure within the heavy lines is that of a square decimetre. A cube, one of whose sides is this area, is a cubic decimetre or *litre*. A litre of water at the temperature of 4° C. weighs a *kilogramme*. A litre of air at 0° C. and 760^{mm} pressure weighs 1·293 gramme.

A litre is 1·76 *pint*; a pint is 0·568 of a litre.

The smaller figures in dotted lines represent the areas of a square centimetre and of a square inch.

A cubic centimetre of water at 4° C. weighs a *gramme*.

Square Inch

Square Centimetre

	Metres	Feet
Millimetre	0·03937	0·003281
Centimetre	0·39371	0·032819
Decimetre	3·93708	0·328090
Metre	39·37079	3·280899
Kilometre	39370·70000	3280·899167

A Hectare or 10,000 square metres is equal to 2·47114 acres, each of which is 43,560 square feet. A kilometre is 0·6214 of a statute mile. A statute mile is 1·609 kilometres. A knot (in telegraphy) is 2,029 yards or 1·1528 statute mile.

Measures of Capacity.

	Cubic Inches	Cubic Feet 1,728 c. in. = 1 c. ft.
Cubic centimetre or millimetre .	0·06103	0·000035
Litre or cubic decimetre . . .	61·02705	0·035317
Kilolitre or cubic metre . .	61,027·05152	35·316581

Measures of Weight.

	English grains	Avoirdupois pounds of 7,000 grains
Milligramme	0·01543	0·0000022
Gramme	15·43235	0·0022046
Kilogramme	15,432·34880	2·2046213

1 grain = 0·064799 gramme; 1 pound avoirdupois is 0·453593 kilogramme.

ELEMENTARY TREATISE

ON

PHYSICS.

BOOK I.

ON MATTER, FORCE, AND MOTION.

CHAPTER I.

GENERAL PRINCIPLES.

1. **Object of Physics.**—The object of *Physics* is the study of the phenomena presented to us by bodies. It should, however, be added, that changes in the nature of the body itself, such as the decomposition of one body into others, are phenomena whose study forms the more immediate object of *chemistry*.

2. **Matter.**—That which possesses the properties whose existence is revealed to us by our senses, we call *matter* or *substance*.

All substances at present known to us may be considered as chemical combinations of sixty-seven *elementary* or *simple* substances. This number, however, may hereafter be diminished or increased by the discovery of some more powerful means of chemical analysis than we at present possess.

3. **Atoms, molecules.**—From various properties of bodies, we conclude that the matter of which they are formed is not perfectly continuous, but consists of an aggregate of an immense number of exceedingly small portions or *atoms* of matter. These atoms cannot be divided physically; they are retained side by side, without touching each other, being separated by distances which are great in comparison with their supposed dimensions.

A group of two or more atoms forms a *molecule*, so that a body may be considered as an aggregate of very small molecules, and these again as aggregates of still smaller atoms. The smallest masses of matter we ever obtain artificially are *particles*, and not molecules or atoms. Molecules retain their position in virtue of the action of certain forces called *molecular forces*.

From considerations based upon various physical phenomena Sir W. Thomson has calculated that in ordinary solids and liquids the average

distance between contiguous molecules is less than the one hundred-millionth but greater than the one two thousand-millionth of a centimetre.

To form an idea of the degree of the size of the molecules Sir W. Thomson gives this illustration:—'Imagine a drop of rain, or a glass sphere the size of a pea, magnified to the size of the earth, the molecules in it being increased in the same proportion. The structure of the mass would then be coarser than that of a heap of fine shot, but probably not so coarse as that of a heap of cricket-balls.'

The number of molecules of gas in a cubic centimetre of air is calculated at twenty-one trillions.

By dissolving in alcohol a known weight of fuchsine, and diluting the liquid, it was observed that a solution containing not more than 0·00000002 of a gramme in one cubic centimetre had still a distinct colour; that is, that a weight of not more than the $\frac{1}{50}$-millionth of a gramme can be perceived by the naked eye. As the molecular weight of this substance is 337 times that of hydrogen it follows that the weight of an atom of hydrogen cannot be greater than the one 20,000-millionth of a gramme.

Loschmidt gives the diameter of the molecules of hydrogen at 0·0000004 of a centimetre; and according to Mousson and Quincke the diameter of the sphere within which one molecule can act upon an adjacent one is between the 0·00006 and 0·00008 of a millimetre, and is therefore from 5 to 10 times less than the wave length of light.

4. **Molecular state of bodies.**—With respect to the molecules of bodies three different stages of aggregation present themselves.

First, the solid state, as observed in wood, stone, metals, &c., at the ordinary temperature. The distinctive character of this state is, that the relative positions of the molecules of the bodies is fixed and cannot be changed without the expenditure of more or less force. As a consequence, solid bodies tend to retain whatever form may have been given to them by nature or by art.

Secondly, the liquid state, as observed in water, alcohol, oil, &c. Here the relative position of the molecules is no longer fixed, the molecules glide past each other with the greatest ease, and the body assumes with readiness the form of any vessel in which it may be placed.

Thirdly, the gaseous state, as in air and in hydrogen. In gases the mobility of the molecules is still greater than in liquids; but the distinctive character of a gas is its incessant struggle to occupy a greater space, in consequence of which a gas has neither an independent form nor an independent volume, for this is due to the pressure to which it is subject.

The general term *fluid* is applied to both liquids and gases.

Most simple bodies, and many compound ones, may be made to pass successively through all the three states. Water presents the most familiar example of this. Sulphur, iodine, mercury, phosphorus, and zinc, are other instances.

5. **Physical phenomena, laws, and theories.**—Every change which can happen to a body, mere alteration of its chemical constitution being excepted, may be regarded as a *physical phenomenon*. The fall of a stone, the vibration of a string, and the sound which accompanies it, the attraction of light particles by a rod of sealing-wax which has been rubbed by flannel,

the rippling of the surface of a lake, and the freezing of water, are examples of such phenomena.

A *physical law* is the constant relation which exists between any phenomenon and its cause. As an example, we have the phenomenon of the diminution of the volume of a gas by the application of pressure; the corresponding law has been discovered, and is expressed by saying that *the volume of a gas is inversely proportional to the pressure.*

In order to explain the cause of whole classes of phenomena, suppositions, or *hypotheses*, are made use of. The utility and probability of a hypothesis or theory are the greater the simpler it is, and the more varied and numerous are the phenomena which are *explained* by it; that is to say, are brought into regular causal connection among themselves and with other natural phenomena. Thus the adoption of the undulatory theory of light is justified by the simple and unconstrained explanation it gives of all luminous phenomena, and by the connection it reveals with the phenomena of heat.

6. **Physical agents.**—In our attempts to ascend from a phenomenon to its cause, we assume the existence of *physical agents*, or *natural forces* acting upon matter; as examples of such we have *gravitation, heat, light, magnetism*, and *electricity*.

Since these physical agents are disclosed to us only by their effects, their intimate nature is completely unknown. In the present state of science, we cannot say whether they are properties inherent in matter, or whether they result from movements impressed on the mass of subtile and imponderable forms of matter diffused through the universe. The latter hypothesis is, however, generally admitted. This being so, it may be further asked, are there several distinct forms of imponderable matter, or are they in reality but one and the same? As the physical sciences extend their limits, the opinion tends to prevail that there is a subtile, imponderable, and eminently elastic fluid called the *ether* distributed through the entire universe; it pervades the mass of all bodies, the densest and most opaque, as well as the lightest or the most transparent. It is also considered that the ultimate particles of which matter is made up are capable of definite motions varying in character and velocity, and which can be communicated to the ether. A motion of a particular kind communicated to the ether can give rise to the phenomenon of heat; a motion of the same kind, but of greater velocity, produces light; and it may be that a motion different in form or in character is the cause of electricity. Not merely do the atoms of bodies communicate motion to the atoms of the ether, but this latter can impart it to the former. Thus the atoms of bodies are at once the sources and the recipients of the motion. All physical phenomena, referred thus to a single cause, are but transformations of motion.

CHAPTER II.

GENERAL PROPERTIES OF BODIES.

7. **Different kinds of properties.**—By the term *properties*, as applied to bodies, we understand the different ways in which bodies present themselves to our senses. We distinguish *general* from *specific* properties. The former are shared by all bodies, and amongst them the most important are *impenetrability*, *extension*, *divisibility*, *porosity*, *compressibility*, *elasticity*, *mobility*, and *inertia*.

Specific properties are such as are observed in certain bodies only, or in certain states of these bodies; such are *solidity*, *fluidity*, *tenacity*, *ductility*, *malleability*, *hardness*, *transparency*, *colour*, &c.

With respect to the above general properties, *impenetrability* and *extension* might, perhaps, be more aptly termed essential attributes of matter, since they suffice to define it; and that *divisibility*, *porosity*, *compressibility*, and *elasticity* do not apply to atoms, but only to bodies or aggregates of atoms (3).

8. **Impenetrability.**—*Impenetrability* is the property in virtue of which two portions of matter cannot at the same time occupy the same portion of space. Thus when a stone is placed in a vessel of water the volume of the water rises by an amount depending on the volume of the stone; this method, indeed, is used to determine the bulk of irregularly shaped bodies by means of graduated measures.

Strictly speaking, this property applies only to the atoms of a body. In many phenomena bodies appear to penetrate each other; thus, the volume of a compound body is always less than the sum of the volumes of its constituents; for instance, the volume of a mixture of water and sulphuric acid, or of water and alcohol, is less than the sum of the volumes before mixture. In all these cases, however, the penetration is merely apparent, and arises from the fact that in every body there are interstices or spaces unoccupied by matter (13).

9. **Extension.**—*Extension* or *magnitude* is the property in virtue of which every body occupies a limited portion of space.

Many instruments have been invented for measuring linear extension or lengths with great precision. Two of these, the vernier and micrometer screw, on account of their great utility, deserve to be here mentioned.

10. **Vernier.**—The *vernier* forms a necessary part of all instruments where lengths or angles have to be estimated with precision; it derives its name from its inventor, a French mathematician, who died in 1637, and consists essentially of a short graduated scale, *ab*, which is made to slide along a fixed scale, AB, so that the graduations of both may be compared

with each other. The fixed scale, AB, being divided into equal parts, the whole length of the vernier, *a b*, may be taken equal to nine of those parts, and is itself divided into ten equal parts. Each of the parts of the vernier, *a b*, will then be less than a part of the scale by one tenth of the latter.

This granted, in order to measure the length of any object, *m n*, let us suppose that the latter, when placed as in the figure, has a length greater than four but less than five parts of the fixed scale. In order to determine by what fraction of a part *m n* exceeds four, one of the ends, *a*, of the vernier, *a b*, is placed in contact with one extremity of the object, *m n*, and the division on the vernier is sought which coincides with a division on the scale, A B. In the figure this coincidence occurs at the eighth division of the vernier, counting from the end, *n*, and indicates that the fraction to be measured is equal to $\frac{8}{10}$ths of a part of the scale, A B. In fact, each of the parts of the vernier being less than a part of the scale by $\frac{1}{10}$th of the latter, it is clear that on proceeding towards the left from the point of coincidence, the divisions of the vernier are respectively one, two, three, etc.

Fig. 1.

tenths behind the divisions of the scale; so that the end, *n*, of the object (that is to say, the eighth division of the vernier) is $\frac{8}{10}$ths behind the division 4 on the scale; in other words, the length of *m n* is equal to $4\frac{8}{10}$ths of the parts into which the scale A B is divided. Consequently, if the scale A B were divided into inches, the length of *m n* would be $4\frac{8}{10} = 4\frac{4}{5}$ inches. The divisions on the scale remaining the same, it would be necessary to increase the length of the vernier in order to measure the length *m n* more accurately. For instance, if the length of the vernier were equal to nineteen of the parts on the scale, and this length were divided into twenty equal parts, the length *m n* could be determined to the twentieth of a part on the scale, and so on. In instruments like the theodolite, intended for measuring angles, the scale and vernier have a circular form, and the latter usually carries a magnifier in order to determine with greater precision the coincident divisions of vernier and scale.

11. **Micrometer screw.**—Another useful little instrument for measuring small lengths with precision is the *micrometer screw*. It is used under various forms, but the principle is the same in all, and may be illustrated by reference to the *spherometer*. This consists of an accurately turned screw with a blunt point which works in a companion supported on three steel points (fig. 2). To one of these is fixed a vertical graduated scale, each division of which is equal to the distance between two threads of the screw.

This distance may be accurately determined by measuring a given length of the screw by compasses, and counting the number of the threads in this length. A milled head attached to the screw is graduated at the periphery into any given number of parts, say 500. Suppose now the distance between the threads is 1 millimetre, when the head has made a complete turn it will have risen or sunk through one millimetre, and so on in proportion for any multiple or fraction of a turn.

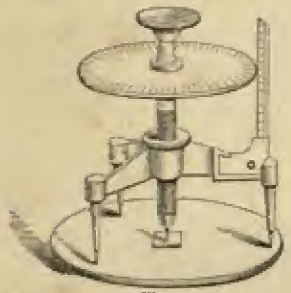

Fig. 2.

In order to determine the thickness of a piece of glass for instance, the apparatus is placed on a perfectly plane polished surface, and the point of the screw is brought in contact with the glass. The division on the vertical scale immediately above the limb, and that on the limb are read off. After removing the glass plate the point is brought in contact with the plane surface, and corresponding readings are again made, from which the thickness can be at once deduced.

The same process is obviously applicable to determining the diameter of a wire.

To ascertain whether a surface is spherical, three points are applied to the surface, and the screw is also made to touch as described above. It is then moved along the surface, and if all four points are everywhere in contact the surface is truly spherical. This application is of great value in ascertaining the exact curvature of lenses.

The diameter of a sphere may also be measured by its means; for it can be shown by a simple geometrical construction that the distance of the movable point from the plane of the fixed points, multiplied by the diameter of the sphere, is equal to the square of the distance of the movable point from one of the fixed points.

12. **Divisibility**—is the property in virtue of which a body may be separated into distinct parts.

Numerous examples may be cited of the extreme divisibility of matter. (3.) The tenth part of a grain of musk will continue for years to fill a room with its odoriferous particles, and at the end of that time will scarcely be diminished in weight. Blood is composed of red, flattened globules, floating in a colourless liquid called *serum*. In man the diameter of one of these globules is less than the 3,500th part of an inch, and the drop of blood which might be suspended from the point of a needle would contain about a million of globules.

Again, the microscope has disclosed to us the existence of insects smaller even than these particles of blood; the struggle for existence reaches even to these little creatures, for they devour still smaller ones. If blood runs in the veins of these devoured ones, how infinitesimal must be the magnitude of its component globules!

Although experiment fails to determine whether there be a limit to the divisibility of matter, many facts in chemistry, such as the invariability in the relative weights of the elements which combine with each other, would

lead us to believe that such a limit does exist. It is on this account that bodies are conceived to be composed of extremely minute and indivisible parts called *atoms* (3).

13. **Porosity.**—*Porosity* is the quality in virtue of which interstices or *pores* exist between the molecules of a body.

Two kinds of pores may be distinguished: *physical pores*, where the interstices are so small that the surrounding molecules remain within the sphere of each other's attracting or repelling forces; and *sensible pores*, or actual cavities across which these molecular forces cannot act. The contractions and expansions resulting from variations of temperature are due to the existence of physical pores, whilst in the organic world the sensible pores are the seat of the phenomena of exhalation and absorption.

In wood, sponge, and a great number of stones—for instance, pumice stone—the sensible pores are apparent; physical pores never are. Yet, since the volume of every body may be diminished, we conclude that all possess physical pores.

The existence of sensible pores may be shown by the following experiment:—A long glass tube, A (fig. 3), is provided with a brass cup at the top, and a brass foot made to screw on to the plate of an air-pump. The bottom of the cup consists of a thick piece of leather. After pouring mercury into the cup so as entirely to cover the leather, the air-pump is put in action, and a partial vacuum produced within the tube. By so doing a shower of mercury is at once produced within the tube, for the atmospheric pressure on the mercury forces that liquid through the pores of the leather. In the same manner water or mercury may be forced through the pores of wood, by replacing the leather in the above experiment by a disc of wood cut perpendicular to the fibres.

Fig. 3.

When a piece of chalk is thrown into water, air-bubbles at once rise to the surface, in consequence of the air in the pores of the chalk being expelled by the water. The chalk will be found to be heavier after immersion than it was before, and from the increase of its weight the volume of its pores may be easily determined.

The porosity of gold was demonstrated by the celebrated Florentine experiment made in 1661. Some academicians at Florence, wishing to try whether water was compressible, filled a thin globe of gold with that liquid, and, after closing the orifice hermetically, they exposed the globe to pressure with a view of altering its form, knowing that any alteration in form must be accompanied by a diminution in volume.

The consequence was, that the water forced its way through the pores of the gold, and stood on the outside of the globe like dew. More than twenty years previously the same fact was demonstrated by Francis Bacon by means of a leaden sphere; the experiment has since been repeated with globes of other metals, and similar results obtained.

14. **Apparent and real volumes.**—In consequence of the porosity of bodies, it becomes necessary to distinguish between their real and apparent volumes. The *real volume* of a body is the portion of space actually occupied by the matter of which the body is composed; its *apparent volume* is the sum of its real volume and the total volume of its pores. The real volume of a body is invariable, but its apparent volume can be altered in various ways.

15. **Applications.**—The property of porosity is utilised in filters of paper, felt, stone, charcoal, &c. The pores of these substances are sufficiently large to allow liquids to pass, but small enough to arrest the passage of any substances which these liquids may hold in suspension. Again, large blocks of stone are often detached in quarries by introducing wedges of dry wood into grooves cut in the rock. These wedges being moistened, water penetrates their pores, and causes them to swell with considerable force. Dry cords, when moistened, increase in diameter and diminish in length—a property of which advantage has been taken in order to raise great weights.

16. **Compressibility.**—*Compressibility* is the property in virtue of which the volume of a body may be diminished by pressure. This property is at once a consequence and a proof of porosity.

Bodies differ greatly with respect to compressibility. The most compressible bodies are gases; by sufficient pressure they may be made to occupy ten, twenty, or even some hundred times less space than they do under ordinary circumstances. In most cases, however, there is a limit beyond which, when the pressure is increased, they become liquids.

The compressibility of solids is much less than that of gases, and is found in all degrees. Cloths, paper, cork, woods, are amongst the most compressible. Metals are so also to a great extent, as is proved by the process of coining, in which the metal receives the impression from the die. There is, in most cases, a limit beyond which, when the pressure is increased, bodies are fractured or reduced to powder.

The compressibility of liquids is so small as to have remained for a long time undetected: it may, however, be proved by experiment, as will be seen in the chapter on Hydrostatics.

17. **Elasticity.**—*Elasticity* is the property in virtue of which bodies resume their original form or volume, when the force which altered that form or volume ceases to act. Elasticity may be developed in bodies by pressure, by traction or *pulling*, flexion or *bending*, and by torsion or *twisting*. In treating of the general properties of bodies, the elasticity developed by pressure alone requires consideration; the other kinds of elasticity, being peculiar to solid bodies, will be considered amongst their specific properties (arts. 89, 90, 91).

Gases and liquids are perfectly elastic; in other words, after undergoing a change in volume they regain exactly their original volume when the pressure becomes what it originally was. Solid bodies present different de-

grees of elasticity, though none present the property in the same perfection as liquids and gases, and in all it varies according to the time during which the body has been exposed to pressure. Caoutchouc, ivory, glass, and marble possess considerable elasticity; lead, clay, and fats, scarcely any.

There is a limit to the elasticity of solids, beyond which they either break or are incapable of regaining their original form and volume. This is called the *limit of elasticity*; within this limit all substances are perfectly elastic. In sprains, for instance, the elasticity of the tendons has been exceeded. In gases and liquids, on the contrary, no such limit can be reached; they always regain their original volume when the original pressure is restored.

If a ball of ivory, glass, or marble be allowed to fall upon a slab of polished marble, which has been previously slightly smeared with oil, it will rebound and rise to a height nearly equal to that from which it fell. On afterwards examining the ball a circular blot of oil will be found upon it, more or less extensive according to the height of the fall. From this we conclude that at the moment of the shock the ball was flattened, and that its rebound was caused by the effort to regain its original form.

18. **Mobility, motion, rest.**—*Mobility* is the property in virtue of which the position of a body in space may be changed.

Motion and rest may be either relative or absolute. By the *relative motion* or *rest* of a body we mean its change or permanence of position with respect to surrounding bodies; by its *absolute motion* or *rest* we mean the change or permanence of its position with respect to ideal fixed points in space.

Thus a passenger in a railway carriage may be in a state of relative rest with respect to the train in which he travels, but he is in a state of relative motion with respect to the objects, such as trees, houses, &c., past which the train rushes. These houses again enjoy merely a state of relative rest, for the earth itself which bears them is in a state of incessant relative motion with respect to the celestial bodies of our solar system, inasmuch as it moves at the rate of more than eighteen miles in a second. In short, absolute motion and rest are unknown to us; in nature, relative motion and rest are alone presented to our observation.

19. **Inertia.**—*Inertia* is a purely negative though universal property of matter (26); it is the property that matter cannot of itself change its own state of motion or of rest. If a body is at rest it remains so until some force acts upon it; if it is in motion this motion can only be changed by the application of some force.

This property of inertia is what is expressed by Newton's first law of motion.

A body, when unsupported in mid-air, does not fall to the earth in virtue of any inherent property, but because it is acted upon by the force of gravity. A billiard ball gently pushed does not move more and more slowly, and finally stop, because it has any preference for a state of rest, but because its motion is impeded by the friction on the cloth on which it rolls, and by the resistance of the air. If all impeding causes were withdrawn, a body once in motion would continue to move for ever in a straight line with unchanging velocity.

20. **Applications.**—Numerous phenomena may be explained by the inertia of matter. For instance, before leaping a ditch we run towards it, in order that the motion of our bodies at the moment of leaping may add itself to the muscular effort then made.

On descending carelessly from a carriage in motion, the upper part of the body retains its motion, whilst the feet are prevented from doing so by friction against the ground; the consequence is we fall towards the moving carriage. A rider falls over the head of a horse if it suddenly stops. In striking the handle of a hammer against the ground the handle suddenly stops, but the head, striving to continue its motion, fixes itself more firmly on the handle.

By the property of inertia may also be explained the following experiments:—Let a card be placed upon a tumbler, and a shilling on the card; if the edge of the card be smartly flicked with the finger the card is driven away and the coin falls into the tumbler. A gentle push with the finger will move a door on its hinges; but if a pistol bullet be fired against the door it perforates the door without moving it. A clay tobacco pipe, which is suspended by two vertical hairs, may be cut in two by a powerful stroke with a sharp sword without breaking the hairs.

A string which gently applied will raise a weight, snaps at once when a sudden pull is exerted. Substances which explode with great rapidity, such as fulminating mercury, chloride of nitrogen, cannot be used with fire-arms, because there is not sufficient time to transfer the motion to the projectiles, and hence the weapons are burst.

The terrible accidents on our railways are chiefly due to inertia. When the motion of the engine is suddenly arrested the carriages strive to continue the motion they had acquired, and in doing so are shattered against each other. Hammers, pestles, stampers are applications of inertia. So are also the enormous iron fly-wheels, by which the motion of steam-engines is regulated.

CHAPTER III.

ON FORCE, EQUILIBRIUM, AND MOTION.

21. **Measure of time.**—To obtain a proper measure of force it is necessary, as a preliminary, to define certain conceptions which are presupposed in that measure; and, in the first place, it is necessary to define the unit of time. Whenever a *second* is spoken of without qualification it is understood to be a second of *mean solar time*. The exact length of this unit is fixed by the following considerations. The instant when the sun's centre is on an observer's meridian—in other words, the instant of the *transit* of the sun's centre—can be determined with exactitude, and thus the interval which elapses between two successive transits also admits of exact determination, and is called an *apparent* day. The length of this interval differs slightly from day to day, and therefore does not serve as a convenient measure of time. Its *average* length is not open to this objection, and therefore serves as the required measure, and is called a *mean solar day*. The short hand of a common clock would go exactly twice round the face in a mean solar day if it went perfectly. The mean solar day consists of 24 equal parts called *hours*, these of 60 equal parts called *minutes*, and these again of 60 equal parts called *seconds*. Consequently, the second is the 86,400th part of a mean solar day, and is the generally received unit of time.

22. **Measure of space.**—Space may be either *length* or *distance*, which is space of one dimension; *area*, which is space of two dimensions; or *volume*, which is space of three dimensions. In England the standard of length is the British Imperial Yard, which is the distance between two fixed points on a certain metal rod, kept in the Tower of London, when the temperature of the whole rod is 60° F. = 15°·5 C. It is, however, usual to employ as a unit, a *foot*, which is the third part of a yard. In France the standard of length is the *metre*; this is approximately equal to the ten-millionth part of a quadrant of the earth's meridian, that is of the arc from the Equator to the North Pole; it is practically fixed by the distance between two marks on a certain standard rod. The relation between these standards is as follows:—

1 yard = 0·914383 metre.
1 metre = 1·093633 yard.

The unit of length having been fixed, the units of area and volume are connected with it thus: the *unit of area* is the area of a square, one side of which is the unit of length. The *unit of volume* is the volume of a cube, one edge of which is the unit of length. These units in the case of English measures are the square yard (or foot) and the cubic yard (or foot) respectively; in the case of French measures, the square metre and cubic metre respectively. The length of the seconds pendulum, in lat. 45°, which is about that of Milan, is 0·9935m., and thus only differs from a metre by 6·5 millimetres.

23. **Measure of mass.**—Two bodies are said to have equal masses when, if placed in a perfect balance *in vacuo*, they counterpoise each other. Suppose we take lumps of any substance, lead, butter, wood, stone, &c., and suppose that any one of them when placed on the one pan of a balance will exactly counterpoise any other of them when placed on the opposite pan—the balance being perfect and the weighing performed *in vacuo*; this being the case, these lumps are said to have equal masses.

The British unit of mass is the standard pound (avoirdupois), which is a certain piece of platinum kept in the Exchequer Office in London. This unit having been fixed, the mass of a given substance is expressed as a multiple or submultiple of the unit.

It need scarcely be mentioned that many distances are ascertained and expressed in yards which it would be physically impossible to measure directly by a yard measure. In like manner the masses of bodies are frequently ascertained and expressed numerically which could not be placed in a balance and subjected to direct weighing.

24. **Density and relative density.**—If we consider any body or portion of matter, and if we conceive it to be divided into any number of parts having equal volumes, then, if the masses of these parts are equal, in whatever way the division be conceived as taking place, that body is one of *uniform density*. The *density* of such a body is the mass of the *unit of volume*. Consequently, if M denote the mass, V the volume, and D the density of the body, we have

$$M = VD.$$

If now we have an equal volume V of any second substance whose mass is M′ and density D′, we shall have

$$M' = VD'.$$

Consequently, $D : D' :: M : M'$; that is, the densities of substances are in the same ratio as the masses of equal volumes of those substances. If now we take the density of distilled water at 4° C. to be unity, the relative density of any other substance is the ratio which the mass of any given volume of that substance at that temperature bears to the mass of an equal volume of water. Thus it is found that the mass of any volume of platinum is 22·069 times that of an equal volume of water, consequently the relative density of platinum is 22·069.

The relative density of a substance is generally called its *specific gravity*. Methods of determining it are given in Book III.

In French measures the *cubic decimetre* or *litre* of distilled water at 4° C. contains the unit of mass, the *kilogramme*; and therefore the mass in kilogrammes of V cubic decimetres of a substance whose specific gravity is D, will be given by the equation

$$M = VD.$$

The same equation will give the mass in *grammes* of the body, if V is given in cubic *centimetres*.

It has been ascertained that 27·7274 cubic inches of distilled water at the temperature of 15°·5 C. or 60° F. contain a pound of matter. Consequently,

if V is the *volume* of a body in cubic inches, D its *specific gravity*, its mass M in pounds avoirdupois will be given by the equation

$$M = \frac{VD}{27 \cdot 7274}.$$

In this equation D is, properly speaking, the relative density of the substance at $15^\circ \cdot 5$ C. when the density of water at $15^\circ \cdot 5$ C. is taken as the unit.

25. **Velocity and its measure.**—When a material point moves, it describes a continuous line which may be either straight or curved, and is called its *path* and sometimes its *trajectory*. Motion which takes place along a straight line is called *rectilinear* motion; that which takes place along a curved line is called *curvilinear* motion. The rate of the motion of a point is called its *velocity*. Velocity may be either uniform or variable; it is *uniform* when the point describes equal spaces or portions of its path in all equal times; it is *variable* when the point describes unequal portions of its path in any equal times.

Uniform velocity is measured by the number of units of space described in a given unit of time. The units commonly employed in this country are feet and seconds. If, for example, a velocity 5 is spoken of without qualification, this means a velocity of 5 feet per second. Consequently, if a body moves for t seconds with a uniform velocity v, it will describe vt feet.

The following are a few examples of different degrees of velocity expressed in this manner. A snail 0·005 feet in a second; the Rhine between Worms and Mainz 3·3; military quick step 4·6; moderate wind 10; fast sailing vessel 18·0; Channel steamer 22·0; railway train 36 to 75 feet; racehorse and storm 50 feet; eagle 100 feet; carrier pigeon 120 feet; a hurricane 160 feet; sound at 0° 1,090; a shot from an Armstrong gun 1,180; a Martini-Henry rifle bullet 1,330; a point on the Equator in its rotation about the earth's axis 1,520; velocity of the vibratory motion of particles of air 1,590; the centre of the earth 101,000 feet; light, and also electricity in a medium destitute of resistance 192,000 miles.

Variable velocity is measured at any instant by the number of units of space a body would describe if it continued to move uniformly from that instant for a unit of time. Thus, suppose a body to run down an inclined plane, it is a matter of ordinary observation that it moves more and more quickly during its descent; suppose that at any point it has a velocity 15, this means that at that point it is moving at the rate of 15 ft. per second, or in other words, if from that point all increase of velocity ceased, it would describe 15 ft. in the next second.

26. **Force.**—When a material point is at rest, it has no innate power of changing its state of rest; when it is in motion it has no innate power of changing its state of uniform motion in a straight line. This property of matter is termed its *inertia* (19). Any cause which sets a point in motion, or which changes the magnitude or direction of its velocity if in motion, is a *force*. *Gravity*, *friction*, the *elasticity* of springs or gases, *electrical* or *magnetic attraction* or *repulsion*, &c., are *forces*. All changes observed in the motion of bodies can be referred to the action of one or more forces.

According to the length of time during which it acts, a force may be either *momentary*—such as the forces called into play in an explosion, an impact, or the discharge of an electrical spark—or it may be *continuous* and

permanent, like the attraction of a magnet or of gravitation, or the forces called into play by an electrical current. The effect of a force of the former kind (which is called an *impulsive* force) is, as far as our observation permits, an instantaneous change in the momentum (28) of the body on which it acts, while the effects of forces of the latter kind are produced gradually, and require the lapse of time to exhibit themselves. In order that impulsive forces may produce any appreciable effects, their intensity during the moment of their action must be indefinitely greater than that of continuous forces. An impulsive force is measured by the instantaneous change in the momentum of the body on which it acts. If the strength of a continuous force does not vary, it is called a *constant* force.

27. **Accelerative effect of force.**—If we suppose a force to continue unchanged in magnitude, and to act along the line of motion of a point, it will communicate in each successive second a constant increase of velocity. This constant increase is the *accelerative effect of the force.* Thus, if at any given instant the body has a velocity 10, and if at the end of the first, second, third, &c., second from that instant its velocity is 13, 16, 19, &c., the accelerative effect of the force is 3; a fact which is expressed by saying that the body has been acted on by an accelerating force 3.

If the force vary from instant to instant, its accelerative effect will also vary; when this is the case the accelerative effect at any instant is measured by the velocity it would communicate in a second if the force continued constant from that instant.

By means of an experiment to be described below (80) it can be shown that at any given place the accelerative effect of gravity g is constant; but it is found to have different values at different places; adopting the units of feet and seconds it is found that with sufficient approximation

$$g = f\,(1 - 0{\cdot}00256 \cos 2\phi)$$

at a place whose latitude is ϕ, where f denotes the number $32{\cdot}1724$, that is the effect of gravity in latitude $45°$.

If we adopt the units of metres and seconds, then $f = 9{\cdot}8059$.

28. **Momentum** or quantity of motion is a magnitude varying as the mass of a body and its velocity jointly, and is therefore expressed numerically by the product of the number of units of mass which it contains, m, and the number of units of velocity, v, in its motion, or by mv. Thus a body containing 5 lbs. of matter, and moving at the rate of 12 ft. per second, has a momentum of 60.

29. **Measure of force.**—Force, when constant, is measured by the *momentum* it communicates to a body in a unit of time. If the force varies, it is then measured at any instant by the momentum it would communicate if it continued constant for a unit of time from the instant under consideration. On the British system of weights and measures the *unit of force* is that force which acting on a pound of matter would produce in one second a velocity of one foot per second. To this unit the term *poundal* has been applied. Consequently, if a body contains m lbs. of matter, and is acted on by a force whose accelerative effect is f, that force contains a number of units of force (F), given by the equation

$$F = mf.$$

The weight of a body, *when that term denotes a force*, is the force exerted on it by gravity; consequently, if m is the mass of the body, and g the accelerating force of gravity, the number of units of force W exerted on it by gravity is given by the equation

$$W = mg$$

or (27) $$W = mf(1 - 0{\cdot}00256 \cos 2\phi).$$

From this it is clear that the weight of the same body will be different at different parts of the earth's surface; this could be verified by attaching a piece of platinum (or other metal) to a delicate spring, and noting the variations in the length of the spring during a voyage from a station in the Northern Hemisphere to another in the Southern Hemisphere—for instance, from London to the Cape of Good Hope.

When, therefore, a *pound* is used as a unit of force it must be understood to mean the force W exerted by gravity on a pound of matter in London. Now, in London, the latitude of which is 51° 30′, the numerical value of g is 32·1912, so that

$$W = 1 \times 32{\cdot}1912\,;$$

in other words, when a pound is taken as the unit of force it contains 32·1912 units of force according to the measure given above. It will be observed that a pound of matter is a completely determinate quantity of matter irrespective of locality, but gravity exerts on a pound of matter a pound (or 32·1912 units) of force at London and other places in about the same latitude as London only; this ambiguity in the term *pound* should be carefully noticed by the student; the context in any treatise will always show in which sense the term is used. The absolute unit of force as defined above is constant; it is about equal to a weight of half an ounce at London.

30. **Representation of forces.**—Draw any straight line AB (fig. 4), and fix on any point O in it. We may suppose a force to act on the point O, along the line AB, either towards A or B: then O is called the *point of application* of the force, AB its line of action; if it acts towards A, its *direction* is OA, if toward B, its direction is OB. It is rarely necessary to make the distinction between the line of action and direction of a force: it being very convenient to make the convention that the statement—a force acts on a point O along the line OA—means that it acts from O to A. Let us suppose the force which acts on O along OA to contain P units of force; from O towards A measure ON containing P units of length, the line ON is said to *represent* the force. The analogy between the line and the force is very complete; the line ON is drawn from O in a given direction OA, and contains a given number of units P, just as the force acts on O in the direction OA, and contains a given number of units P. It is scarcely necessary to add, that if an equal force were to act on O in the opposite direction, it would be said to act in the direction OB, and would be represented by OM, equal in magnitude to ON.

Fig. 4

When we are considering several forces acting along the same line we may indicate their directions by the positive and negative signs. Thus the forces mentioned above would be denoted by the symbols + P and − P respectively.

31. **Forces acting along the same line.**—If forces act on the point O in the direction OA equal to P and Q units respectively, they are equivalent to a single force R containing as many units as P and Q together; that is,

$$R = P + Q.$$

If the sign + in the above equation *denote algebraical* addition, the equation will continue true whether one or both the forces act along OA or OB. It is plain that the same rule can be extended to any number of forces, and if several forces have the same line of action they are equivalent to one force containing the same number of units as their *algebraical* sum. Thus if forces of 3 and 4 units act on O in the direction OA, and a force of 8 in the direction OB, they are equivalent to a single force containing R units given by the equation

$$R = 3 + 4 - 8 = -1;$$

that is, R is a force containing one unit acting along OB. This force R is called their *resultant*. If the forces are in equilibrium R is equal to zero. In this case the forces have equal tendencies to move the point O in opposite directions.

32. **Resultant and components.**—In the last article we saw that a single force R could be found equivalent to several others; this is by no means peculiar to the case in which all the forces have the same line of action; in fact, when a material point, A (fig. 5), remains in equilibrium under the action of several forces, S, P, Q, it does so because any one of the forces, as S, is capable of neutralising the combined effects of all the others. If the force S, therefore, had its direction reversed, so as to act along A R, the prolongation of A S, it would produce the same effect as the system of forces P, Q.

Fig. 5.

Now, a force whose effect is equivalent to the combined effects of several other forces is called their *resultant*, and, with respect to this resultant, the other forces are termed *components*.

When the forces P, Q act on a point they can only have *one* resultant; but any single force can be resolved into components in an indefinite number of ways.

If a point move from rest, under the action of any number of forces, it will begin to move in the direction of their resultant.

33. **Parallelogram of forces.**—When two forces act on a point their resultant is found by the following theorem, known as the principle of the parallelogram of forces:—*If two forces act on a point, and if lines be drawn from that point representing the forces in magnitude and direction, and on these lines as sides a parallelogram be constructed, their resultant will be represented in magnitude and direction by that diagonal which passes through the point.* Thus let P and Q (fig. 6) be two forces acting on the point A along AP and AQ respectively, and let AB and AC be taken containing the same number of units of length that P and Q contain units of force; let the parallelogram ABDC be completed, and the diagonal AD drawn; then the theorem states that the resultant, R, of P and Q is represented by AD; that is to say, P and Q together are equal to a single force R acting along the

line AD, and containing as many units of force as AD contains units of length.

Proofs of this theorem are given in treatises on Mechanics; we will here give an account of a direct experimental verification of its truth; but before doing so we must premise an account of a very simple experiment.

Let A (fig. 7) be a small pulley, and let it turn on a smooth, hard, and thin axle with little or no friction; let W be a weight tied to the end of a fine thread which passes over the pulley; let a spring CD be attached by one end to the end C of the thread and by the end D to another piece of thread, the other end of which is fastened to a fixed point B; a scale CE can be fastened by one end to the point C and pass inside the spring so that the elongation of the spring can be measured. Now it will be found on trial that with a given weight W the elongation of the spring will be the same whatever the angle contained between the parts of the string WA and BA. Also it would be found that if the whole were suspended from a fixed point, instead of passing over the pulley, the weight would in this case stretch the spring to the same extent as before. This experiment shows that when care is taken to diminish to the utmost the friction of the axle of the pulley, and the imperfect flexibility of the thread, the weight of W is transmitted without sensible diminution to B, and exerts on that point a pull or force along the line BA virtually equal to W.

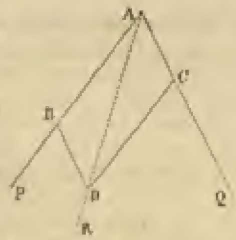

Fig. 6.

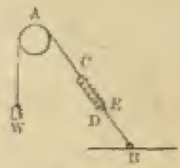

Fig. 7.

This being premised, an experimental proof, or illustration of the parallelogram of forces, may be made as follows:—

Suppose H and K (fig. 8) to be two pulleys with axles made as smooth and fine as possible; let P and Q be two weights suspended from fine and flexible threads which, after passing over H and K, are fastened at A to a third thread AL from which hangs a weight R; let the three weights come to rest in the positions shown in the figure. Now the point A is acted on by three forces in equilibrium, viz. P from A to H, Q from A to K, and R from A to L; consequently, any one of them must be equal and opposite to the resultant of the other two. Now if we suppose the apparatus to be arranged immediately in front of a large slate, we can draw lines upon it coinciding with AH, AK, and AL. If now we measure off along AH the part AB containing as many inches as P contains pounds, and along AK the part AC containing as many inches as Q contains pounds, and complete the parallelogram ABCD, it will be found that the diagonal AD is in the same line as AL, and contains as many inches as R weighs pounds. Consequently, the resultant of P and Q is represented by AD. Of course, any other units of length and force might

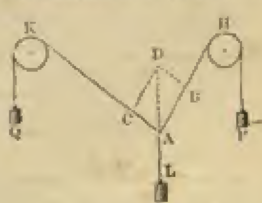

Fig. 8.

have been employed. Now it will be found that when P, Q, and R are changed in any way whatever, consistent with equilibrium, the same construction can be made,—the point A will have different positions in the different cases; but when equilibrium is established, and the parallelogram ABCD is constructed, it will be found that AD is vertical, and contains as many units of length as R contains units of force, and consequently it represents a force equal and opposite to R; that is, it represents the resultant of P and Q.

34. **Resultant of any number of forces acting in one plane on a point.**—Let the forces P, Q, R, S (fig. 9) act on the point A, and let them be represented by the lines AB, AC, AD, AE, as shown in the figure. *First*, complete the parallelogram ABFC and join AF; this line represents the resultant of P and Q. *Secondly*, complete the parallelogram AFGD and join AG; this line represents the resultant of P, Q, R. *Thirdly*, complete the parallelogram AGHE and join AH; this line represents the resultant of P, Q, R, S. It is manifest that the construction can be extended to any number of forces. A little consideration will show that the line AH might be determined by the following construction:—Through B draw BF parallel to, equal to, and towards the same part as AC; through F draw FG parallel to, equal to, and towards the same part as AD; through G draw GH parallel to, equal to, and towards the same part as AE; join AH, then AH represents the required resultant.

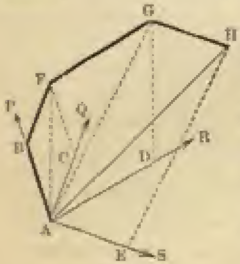

Fig. 9.

In place of the above construction, the resultant can be determined by calculation in the following manner:—Through A draw any two rectangular axes AX and AY (fig. 10), and let α, β, γ be the angles made with the axis AX by the lines representing the pressures, then P, Q, R can be resolved into P cos α, Q cos β, R cos γ, acting along AX, and P sin α, Q sin β, R sin γ, acting along AY. Now the former set of forces can be reduced to a single force X by addition, attention being paid to the sign of each component; and in like manner the latter forces can be reduced to a single force Y, that is,

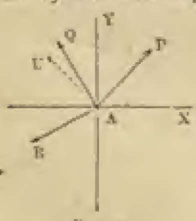

Fig. 10.

$$X = P\cos\alpha + Q\cos\beta + R\cos\gamma + \ldots$$
$$Y = P\sin\alpha + Q\sin\beta + R\sin\gamma + \ldots$$

Since the addition denotes the *algebraical* sum of the quantities on the right-hand side of the equations, both *sign* and *magnitude* of X and Y are known. Suppose U to denote the required resultant, and ϕ the angle made, by the line representing it, with the axis AX;

then $$U\cos\phi = X, \text{ and } U\sin\phi = Y.$$

These equations give $U^2 = X^2 + Y^2$, which determines the magnitude of the resultant, and then, since both sin ϕ and cos ϕ are known, ϕ is determined without ambiguity.

Thus let P, Q, and R be forces of 100, 150, and 120 units, respectively,

and suppose XAP, XAQ, and XAR to be angles of 45°, 120°, and 210° respectively. Then their components along Ax are 70·7, −75, −103·9, and their components along AY are 70·7, +129·9, −60. The sums of these two sets being respectively −108·2 and 140·6, we have $U \cos \phi = -108{\cdot}2$ and $U \sin \phi = 140{\cdot}6$;

therefore $$U^2 = (108{\cdot}2)^2 + (140{\cdot}6)^2$$

or $$U = 177{\cdot}4$$

hence $$177{\cdot}4 \cos \phi = -108{\cdot}2, \text{ and } 177{\cdot}4 \sin \phi = 104{\cdot}6.$$

If we made use of the former of these equations only, we should obtain ϕ equal to 232° 25′, or 127° 35′, and the result would be ambiguous: in like manner, if we determine ϕ from the second equation only, we should have ϕ equal to 52° 25′, or 127° 35′; but as we have both equations, we know that ϕ equals 127° 35′, and consequently the force U is completely determined as indicated by the dotted line AU.

35. **Conditions of equilibrium of any forces acting in one plane on a point.**—If the resultant of the forces is zero, they have no joint tendency to move the point, and consequently are in equilibrium. This obvious principle enables us to deduce the following constructions and equations, which serve to ascertain whether given forces will keep a point at rest.

Suppose that in the case represented in fig. 9, T is the force which will balance P, Q, R, S. It is clear that T must act on A along H A produced, and in magnitude must be proportional to H A; for then the resultant of the five forces will equal zero, since the broken line ABFGHA returns to the point A. This construction is plainly equivalent to the following: Let P, Q, R (fig. 11) be forces acting on the point O, as indicated, their magnitudes and directions being given. It is known that they are balanced by a fourth force, S, and it is required to determine the magnitude and direction of S. Take any point D, and draw any line parallel to and towards the same part as O P, draw A B parallel to and towards the same part as O Q, and take A B such that P : Q :: D A : A B. Through B draw B C parallel to and towards the same part as O R, taking B C such that Q : R :: A B : B C; join C D; through O draw O S parallel to and towards the same part as C D, then the required force acts along O S, and is in magnitude proportional to C D.

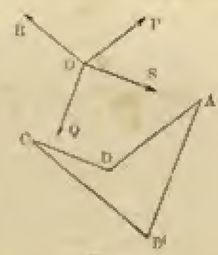

Fig. 11.

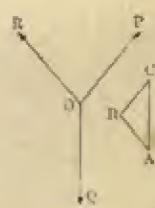

Fig. 12.

It is to be observed that this construction can be extended to any number of forces, and will apply to the case in which these directions are not in one plane, only in this case the broken line A B C D would not lie wholly in one plane. The above construction is frequently called the *Polygon of Forces.*

The case of three forces acting on a point is, of course, included in the above; but its importance is such that we may give a separate statement of

it. Let P, Q, R (fig. 12) be three forces in equilibrium on the point O. From any point B draw BC parallel to and towards the same part OP, from C draw CA parallel to and towards the same part as OQ, and take CA such that P : Q :: BC : CA; then, on joining AB, the third force R must act along OR parallel to and towards the same part as AB, and must be proportional in magnitude to AB. This construction is frequently called the *Triangle of Forces.* It is evident that while the sides of the triangle are severally proportional to P, Q, R, the angles A, B, C are supplementary to QOR, ROP, POQ respectively; consequently, every trigonometrical relation existing between the sides and angles of ABC will equally exist between the forces P, Q, R, and the supplements of the angles between their directions. Thus in the triangle ABC it is known that the sides are proportional to the sines of the opposite angles; now, since the sines of the angles are equal to the sines of their supplements, we at once conclude that *when three forces are in equilibrium, each is proportional to the sine of the angle between the directions of the other two.*

We can easily obtain from the equations which determine the resultant of any number of forces (34) equations which express the conditions of equilibrium of any number of forces acting in one plane on a point; in fact, if U = 0 we must have X = 0 and Y = 0; that is to say, the required conditions of equilibrium are these:—

$$0 = P\cos\alpha + Q\cos\beta + R\cos\gamma + \ldots$$

and

$$0 = P\sin\alpha + Q\sin\beta + R\sin\gamma + \ldots$$

The first of these equations shows that no part of the motion of the point can take place along Ax, the second that no part can take place along Ay. In other words, the point cannot move at all.

36. **Composition and resolution of parallel forces.**—The case of the equilibrium of three parallel forces is merely a particular case of the equilibrium of three forces acting on a point. In fact, let P and Q be two forces whose directions pass through the points A and B, and intersect in O; let them be balanced by a third force R whose direction produced intersects the line AB in C. Now suppose the point O to move along AO, gradually receding from A, the magnitude and direction of R will continually change, and also the point C will continually change its position, but will always lie between A and B. In the limit P and Q become parallel forces, acting towards the same part balanced by a parallel force R acting towards the contrary part through a point X between A and B. The question is:—*First*, on this limiting case what is the value of R; *secondly*, what is the position of X? Now with regard to the first point it is plain that if a triangle *abc* were drawn as in art. 35, the angles *a* and *b* in the limit will vanish, and *c* will become 180°, consequently *ab* ultimately equals *ac* + *cb*;

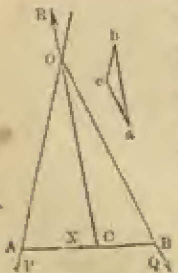

Fig. 13.

or

$$R = P + Q.$$

With regard to the second point it is plain that

$$OC \sin POR = OC \sin AOC = AC \sin CAO$$

and

$$OC \sin ROQ = OC \sin BOC = CB$$

therefore AC sin CAO : CB sin CBO :: sin POR : sin ROQ
:: Q : P (35).

Now in the limit, when OA and OB become parallel, OAB and OBA become supplementary; that is, their sines become equal; also AC and CB become respectively AX and XB; consequently

$$AX : XB :: Q : P,$$

a proportion which determines the position of X. This theorem at once leads to the rules for the composition of any two parallel forces, viz.—

I. When two parallel forces P and Q act towards the same part, at rigidly connected points A and B, their resultant is a parallel force acting towards the same part, equal to their sum, and its direction divides the line AB into two parts AC and CB inversely proportional to the forces P and Q.

II. When two parallel forces P and Q act towards contrary parts at rigidly connected points A and B, of which P is the greater, their resultant is a parallel force acting towards the same part as P, equal to the excess of P over Q, and its direction divides BA produced in a point C such that CA and CB are inversely proportional to P and Q.

In each of the above cases if we were to apply R at the point C, in opposite directions to those shown in the figure, it would plainly (by the above theorem)

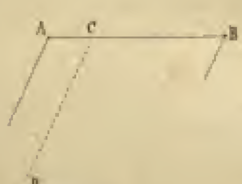

Fig. 14.

Fig. 15.

balance P and Q, and therefore when it acts as shown in figs. 14 and 15 it is the resultant of P and Q in those cases respectively. It will, of course, follow that the force R acting at C can be resolved into P and Q acting at A and B respectively.

If the second of the above theorems be examined, it will be found that no force R exists equivalent to P and Q when these forces are equal. Two such forces constitute *a couple*, which may be defined to be two equal parallel forces acting towards contrary parts; they possess the remarkable property that they are incapable of being balanced by any single force whatsoever.

In the case of more than two parallel forces the resultant of any two can be found, then of that and a third, and so on to any number; it can be shown that however great the number of forces they will either be in equilibrium or will reduce to a single resultant or to a couple.

37. **Centre of parallel forces.**—On referring to figs. 14 and 15, it will be remarked that if we conceive the points A and B to be fixed in the directions

AP and BQ of the forces P and Q, and if we suppose those directions to be turned round A and B, so as to continue parallel and to make any given angles with their original directions, then the direction of their resultant will continue to pass through C; that point is therefore called *the centre* of the parallel forces P and Q.

It appears from investigation, that whenever a system of parallel forces reduces to a single resultant, those forces will have a centre; that is to say, if we conceive each of the forces to act at a fixed point, there will be a point through which the direction of their resultant will pass when the directions of the forces are turned through any equal angles round their points of application in such a manner as to retain the parallelism of their directions.

The most familiar example of a centre of parallel forces is the case in which the forces are the weights of the parts of a body; in this case the forces all acting towards the same part will have a resultant, viz. their sum; and their centre is called the *centre of gravity* of the body.

38. **Moments of forces.**—Let P (fig. 16) denote any force acting from B to P, take A any point, let fall AN a perpendicular from A on BP. The product of the number of units of force in P, and the number of units of length in AN, is called the moment of P with respect to A. Since the force P can be represented by a straight line, the moment of P can be represented by an area. In fact, if BC is the line representing P, the moment is properly represented by twice the area of the triangle ABC. The perpendicular AN is sometimes called the arm of the pressure. Now if a watch were placed with its face upwards on the paper, the force P would cause the arm AN to turn round A in the *contrary* direction to the hands of the watch. Under these circumstances, it is usual to consider the moment of P with respect to the point A to be positive. If P acted from C to B, it would turn NA in the *same* direction as the hands of the watch, and now its moment is reckoned *negative*.

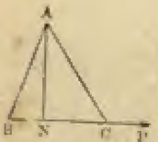

Fig. 16.

The following remarkable relation exists between any forces acting in one plane on a body and their resultant. Take the moments of the forces and of their resultant with respect to any one point in the plane. Then the moment of the resultant equals the sum of the moments of the several forces, regard being had to the *signs* of the moments.

If the point about which the moments are measured be taken in the direction of the resultant, its moment with respect to that point will be zero; and consequently the sum of the moments with respect to such point will be zero.

39. **Equality of action and reaction.**—We will proceed to exemplify some of the principles now laid down by investigating the conditions of equilibrium of bodies in a few simple cases; but before doing so we must notice a law which holds good whenever a mutual action is called into play between two bodies. *Reaction is always equal and contrary to action; that is to say, the mutual actions of two bodies on each other are always forces equal in amount and opposite in direction.* This law is perfectly general, and is equally true when the bodies are in motion as well as when they are at rest. A very instructive example of this law has already been given (33),

in which the action on the spring CD (fig. 7) is the weight W transmitted by the spring to C, and balanced by the reaction of the ground transmitted from B to D. Under these circumstances the spring is said to be stretched by a force W. If the spring were removed, and the thread were continuous from A to B, it is clear that any part of it is stretched by two equal forces, viz. an action and reaction, each equal to W, and the thread is said to sustain a tension W. When a body is urged along a smooth surface, the mutual action can only take place along the common perpendicular at the point of contact. If, however, the bodies are rough, this restriction is partially removed, and now the mutual action can take place in any direction not making an angle greater than some determinate angle with the common perpendicular. This determinate angle has different values for different substances, and is sometimes called the *limiting angle of resistance*, sometimes the *angle of repose*.

40. **The lever** is a name given to any bar straight or curved, AB (fig. 17) resting on a fixed point or edge c called the *fulcrum*. The forces acting on the lever are the *weight* or resistance Q, the *power* P, and the reaction of the fulcrum. Since these are in equilibrium, the resultant of P and Q must act through c, for otherwise they could not be balanced by the reaction. Draw cb at right angles to QB and ca to PA produced; then observing that $P \times ca$, and $Q \times cb$ are the moments of P and Q with respect to c, and that they have contrary signs, we have by (38),

$$P \times ca = Q \times cb;$$

an equation commonly expressed by the rule, that *in the lever the power is to the weight in the inverse ratio of their arms.*

Fig. 17.

Levers are divided into three kinds, according to the position of the fulcrum with respect to the points of application of the power and the weight. In a *lever of the first kind* the fulcrum is between the power and resistance, as in fig. 17, and as in a poker and in the common steelyard; a pair of scissors and a carpenter's pincers are double levers of this kind. In a *lever of the second kind* the resistance is between the power and the fulcrum, as in a wheelbarrow, or a pair of nutcrackers, or a door; in a *lever of the third kind* the power is between the fulcrum and the resistance, as in a pair of tongs or the treadle of a lathe.

41. **Pulleys.**—The pulley is a hard circular disc of wood or of metal, in the edge of which is a groove, and which can turn freely on an axis in the centre. Pulleys are either *fixed*, as in fig. 18, where the stirrup or fork is rigidly connected with some immovable body, and where the axis rotates in the stirrup; or it may be movable, as in fig. 19, where the axis is fixed to the fork, and it passes through a hole in the centre of the disc. The rope which passes round the pulley in fig. 18, supports a weight at one end; while at the other a pull is applied to hold this weight in equilibrium.

We may look upon the power and the resistance as acting at the circumference of the circle; hence as the radii are equal, if we consider the pulley as a lever, the two arms are equal, and equilibrium will prevail when the power and the resistance are equal. The fixed pulley affords thus no mechanical advantage, but is simply convenient in changing the direction of the application of a force.

Fig. 18. Fig. 19.

In the case of the movable pulley the one end of the rope is suspended to a fixed point in a beam, and the weight is attached to the hook on which the pulley acts. The tension of the rope is everywhere the same; one portion of the weight is supported by the fixed part and the other by the power, and these are equal to each other, and are together equal to the weight, including the pulley itself; hence in this case $P = \frac{1}{2} Q$.

If several pulleys are joined together on a common axis in a special sheath, which is fixed, and a rope passes round all those and also round a similar but movable combination of pulleys, such an arrangement, which is represented in fig. 20, is called a *block and tackle.*

If we consider the condition of the rope it will be found to have everywhere the same tension; the weight Q which is attached to the hook common to the whole system is supported by the six portions of the rope; hence each of these portions will sustain one sixth of the weight: the force which is applied at the free end of the rope which passes over the upper pulley, and which determines the tension, will have the same value; that is to say, it will support one sixth of the weight.

The relation between power and resistance in a block and tackle is expressed by the equation $P = \frac{Q}{n}$, in which P is the power, Q the weight, and n the number of cords by which the weight is supported.

42. The wheel and axle.—The older form of this machine, fig. 21, is that of an axle, to which is rigidly fixed, concentric with it, a wheel of larger diameter. The power is applied tangentially on the wheel, and the resistance tangentially to the axle, as for instance in the treadmill and water-wheel. Sometimes, as in the case of the capstan, the power is applied to spokes fixed in the axle, which represent semi-diameters of the wheel; in other cases, as in the windlass, the handle is rigidly fixed to the axis.

In all its modifications we may regard the wheel and axle as an application of the lever, the arms of which are the radii of the wheel and axle respectively, and in all cases equilibrium exists where the power is to the

resistance as the radius of the axle is to the radius of the wheel. Thus in fig. 21, $P : Q = ab : ac$, or $P \times ac = Q \times ab$.

Frequent applications of wheels of different diameters are met with in which the motion of one wheel is transmitted to another, either by means of teeth fitting in each other on the circumference of the wheels, as in fig. 22, or by means of bands passing over the two wheels, as in the illustration of Ladd's Magneto-Electrical Machine (see Book viii.).

In fig. 22, which represents the essential parts of a crab winch, in order to raise the weight Q a power p must be applied at the circumference of the wheel such that $p = Q\frac{r}{R}$, in which r and R are the radii of the axle b and of the toothed wheel a respectively.

The rotation of the wheel a is effected by means of the smaller wheel c or *crab*, the teeth of which fit in those of a. But if this wheel c is to exert at its circumference a power p, the power P which is applied at the end of the handle must be $P = \frac{r'}{R'}p$, in which r' is the radius of c, R' the length of a lever at the end of which P acts, and consequently

$$P = \frac{rr'}{RR'}Q.$$

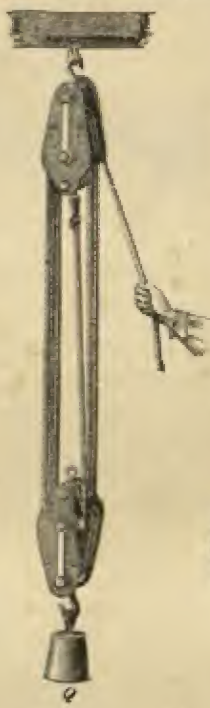

Fig. 20.

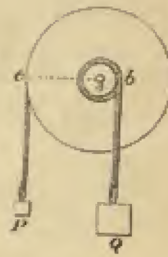

Fig. 21.

Fig. 22.

The radius of the wheel c is to that of the wheel a as their respective circumferences; and, as the teeth of each are of the same size, the circumferences will be as the number of teeth.

Trains of wheelwork are used, not only in raising great weights by the exertion of a small power; as in screw jacks, cranes, crab winches, &c., but also in clock and watch works, and in cases in which changes in velocity or in power or even in direction are required. Numerous examples will be met with in the various apparatus described in this work.

43. **Inclined Plane.**—The properties and laws of the inclined plane may be conveniently demonstrated by means of the apparatus represented in fig. 23. RS represents the section of a smooth piece of hard wood hinged at R; by means of a screw it can be clamped at any angle x against the arc-shaped support, by which at the same time the angle can be measured; a is a cylindrical roller, to the axis of which is attached a string passing over a pulley to a scale-pan P.

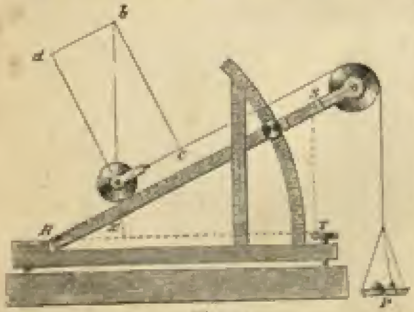

Fig. 23.

It is thus easy to ascertain by direct experiments what weights R must be placed in the pan P in order to balance a roller of any given weight, or to cause it to move with a given angle of inclination.

The line RS represents the *length*, ST the *height*, and RT the *base* of inclined plane.

In ascertaining the theoretical conditions of equilibrium we have a useful application of the parallelogram of forces. Let the line ab, fig. 23, represent the force which the weight W of the cylinder exerts acting vertically downwards; this may be decomposed into two others; one, ad, acting at right angles against the plane, and representing the *pressure* which the weight exerts against the plane; and which is counterbalanced by the reaction of the plane; the other, ac, represents the component which tends to move the weight down the plane, and this component has to be held in equilibrium by the weight, P, equal to it and acting in opposite directions.

It can be readily shown that the triangle abc is similar to the triangle SRT, and that the sides ac and ab are in the same proportion as the sides ST and SR. But the line ac represents the power, and the line ab the weight; hence

$$ST : SR = P : W;$$

that is, on an inclined plane, equilibrium obtains *when the power is to the weight as the height of the inclined plane to its length.*

Since the ratio $\frac{ST}{SR}$ is the sine of the angle x, we may also state the principle thus:

$$P = W \sin x.$$

The component da or bc, which represents the actual pressure against the plane, is equal to $W \cos x$; that is, the pressure against the plane is to the weight, as the base is to the length of the inclined plane.

In the above case it has been considered that the power acts parallel to the inclined plane. It may be applied so as to act horizontally. It will then be seen from fig. 24 that the weight W may be decomposed into two forces, one of which, ab, acts at right angles to the plane, and the other, ac, parallel to the base. It is this latter which is to be kept in equilibrium by the power. From the similarity of the two triangles acb and STR, $ac : bc = ST : TR = h : b$; but bc is equal to W, and ac is equal to P, hence the power which

must be applied at b to hold the weight W in equilibrium is as the height of the inclined plane is to the base, or as the tangent of the angle of inclination x; that is, $P = W \tan x$. The pressure upon the plane in this case may be easily shown to be $ab = \frac{bc}{\cos x}$, that is $= \frac{W}{\cos x}$. This is sometimes called the *relative weight* on the plane.

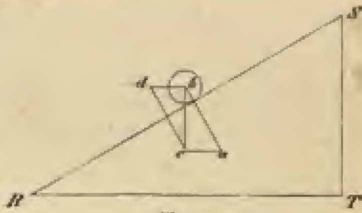

Fig. 24.

If the force P which is to counterbalance W is not parallel to the plane, but forms an angle, E, with it, this force can be decomposed into one which is parallel to it, and one which is at right angles. Of these only the first is operative and is equal to P cos E.

In most cases of the use of the inclined plane, such as in moving carriages and waggons along roads, in raising casks into waggons or warehouses, the power is applied parallel to the inclined plane. An instance of a case in which a force acts parallel to the base is met with in the screw.

Owing to the unevenness of the surfaces in actual use, the laws of equilibrium and of motion on an inclined plane undergo modification. The *friction*, for instance, which comes into play amounts on ordinary roads to from $\frac{1}{15}$ to $\frac{1}{30}$, and on railways to from $\frac{1}{140}$ to $\frac{1}{540}$ of the relative weight. This must be looked upon as a hindrance to be continually overcome, and must be deducted from the force required to keep a body from falling down an inclined plane, or must be added to it in the case in which a body is to be moved up the plane. Hence the use of the inclined plane in unloading heavy casks into cellars, &c.

A body on an inclined plane which cannot rotate does not move provided the inclination is below a certain amount (39). The determination of this *limiting angle of resistance*, at which a body on an inclined plane just begins to move, may serve as a rough illustration of a mode of ascertaining the 'coefficient of friction.'

For in the case in which the power is applied parallel to the plane, the component of the weight which presses against the plane or the actual load, L, is W cos x; and the component which tends to move the body down the plane is equal to W sin x. If the friction, R, is just sufficient to hold this in equilibrium, the coefficient of friction will be $\frac{R}{L} = \frac{W \sin x}{W \cos x} = \tan x$.

Thus if we place on the plane a block of the same material, by gradually increasing the inclination it will begin to move at a certain angle, which will depend on the nature of the material; this angle is the limiting angle of resistance, and its tangent is the coefficient of friction for that material.

44. **The Wedge.**—The ordinary form of the wedge is that of a three-sided prism of iron or steel, one of whose angles is very acute. Its most frequent use is in splitting stone, timber, etc. Fig. 25 represents in section the application of the wedge to this purpose. The side ab is the *back*, the vertex of the angle acb which the two faces ac and bc make with each other represents the *edge*, and the faces ac and bc the *sides* of the wedge. The power P is usually applied at right angles to the back; and we may look

upon the cohesion between the fibres of the wood as representing the resistance to be overcome; as corresponding to what in other machines is the weight. Suppose this to act at right angles to the two faces of the wedge, and to be represented by the lines *fe* and *ge*; complete the parallelogram *gef*, then the diagonal *he* will represent the resultant of the reaction of the fibres tending to force the wedge out; the force which must be applied to hold this wedge in equilibrium must therefore be equal to *eh*. Now *efh* is similar to the triangle *acb*, therefore $ab : ac = eh \,.\, ef$; but these lines represent the pressure applied at the back of the wedge, and the pressure on the face *ac*, hence if P represent the former and Q the latter, there is equilibrium when $P : Q = ab : bc$, that is, when the power is to the resistance in the same ratio as the back of the wedge bears to one of the sides. The relation between power and resistance is more favourable, the sharper the edge, that is, the smaller the angle which the sides make with each other.

Fig. 25.

The action of all sharp cutting instruments, such as chisels, knives, scissors, &c., depends on the principle of the wedge. It is also applied when very heavy weights are to be raised through a short distance, as in launching ships, and in bracing columns and walls to the perpendicular.

45. **The Screw.**—Let us suppose a piece of paper in the shape of a right-angled triangle *aee'* be applied with its vertical side *ac'e'* against a cylinder, and parallel to the axis, and be wrapped round the cylinder; the hypotenuse will describe on the surface of the cylinder a screw line or *helix* (fig. 26); the points *a b c d e* will occupy the positions respectively *a b' c' d' e'*. If the dimensions be so chosen that the base of the triangle *ee'* is equal to the circumference of the cylinder, then the hypotenuse *abc* becomes an inclined plane traced on the surface of the cylinder; the distance *ae'* being the height of the plane.

Fig 26.

An ordinary screw consists of an elevation on a solid cylinder; this elevation may be either square, as in fig. 27, or acute, and such screws are called *square* or *sharp* screws accordingly.

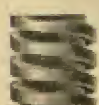
Fig. 27.

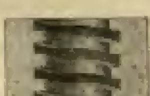
Fig. 28.

When a corresponding groove is cut in the hollow cylinder or out of the same diameter as the bolt, this gives rise to an internal or companion screw or *nut*, fig. 28.

The vertical distance between any two threads of a screw measured parallel to the axis is called the *pitch*, and the angle acc' or ace' is called the *inclination* of the screw.

In practice, a raised screw is used with its companion in such a manner that the elevations of the one fit into, and coincide with, the depressions of the other. The screw is a modification of the inclined plane, and the conditions of equilibrium are those which obtain in the case of the plane. The resistance, which is either a weight to be raised or a pressure to be exerted, acts in the direction of the vertical, and the power acts parallel to the base; hence we have $P : R = h : b$, and the length of the base is the circumference of the cylinder; whence $P : R = h : 2\pi r$; r being the radius of the cylinder, and h the pitch of the screw.

The power is usually applied to the screw by means of a lever, as in the bookbinders' press, &c., and the principle of the screw may be stated to be generally that the power of the screw is to the resistance in the same ratio as that of the pitch of the screw to the circumference of the circle through which the power acts.

46. **Virtual Velocity.**—If the point of application of a force be slightly displaced, the resolved part of the displacement in the direction of the force is termed the *virtual velocity of the force*, and is considered as positive or negative, according as it is in the same direction as the force, or in the opposite direction. Thus, in fig. 29 let the point of application A of the force P be displaced to A′, and draw A′a perpendicular to AP. Then Aa is the virtual velocity of the force P, and being, in this case, in the direction of P, is to be considered positive.

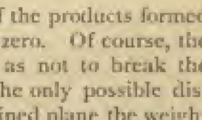

Fig. 29.

The principle of virtual velocities asserts that if any machine or system be kept in equilibrium by any number of forces, and the machine or system then receive any *very small* displacement, the algebraic sum of the products formed by multiplying each force by its virtual velocity will be zero. Of course, the displacement of the machine is supposed to be such as not to break the connection of its parts; thus in the wheel and axle the only possible displacement is to turn it round the fixed axle; in the inclined plane the weight must still continue to rest on the plane; in the various systems of pulleys the strings must still continue stretched, and must not alter in length, &c.

The complete proof of this principle is beyond the scope of the present work, but we may easily establish its truth in any of the machines we have already considered. It will be found in every case that, if the machine receive a small displacement, the virtual velocities of P and W will be of opposite signs, and that, neglecting the signs, P × P's virtual velocity = W × W's virtual velocity. Thus, to take the case of a *bent lever*, let P and Q be the forces acting at the extremities of the arms of the bent lever AFB (fig. 30), and let the lever be turned slightly round its fulcrum F, bringing A to A′, and B to B′. Draw A′a and B′b perpendicular to P and Q respectively; then Aa is the virtual velocity of P, and Bb that of Q, the former being positive and the latter negative. Let Fp, Fq be the perpendiculars from the fulcrum upon P and Q, or what we have called (art. 40) the arms of P and Q. Now, as the displacement is very small, the angles FAA′, FBB′ will be very nearly

right angles; and, therefore, the right-angled triangles AaA', BbB' will ultimately be similar to the triangles FpA, FqB respectively, whence $\frac{Aa}{AA'} = \frac{Fp}{FA'}$, and $\frac{Bb}{BB'} = \frac{Fq}{FB'}$, or $\frac{Aa}{Fp} = \frac{AA'}{FA'}$, and $\frac{Bb}{Fq} = \frac{BB'}{FB}$. But the triangles FAA', FBB' are similar, as they are both isosceles, and their vertical angles are equal, so that $\frac{AA'}{FA} = \frac{BB'}{FB}$; whence $\frac{Aa}{Fp} = \frac{Bb}{Fq}$, or, as we may put it, $\frac{P \times Aa}{P \times Fp} = \frac{Q \times Bb}{Q \times Fq}$. Now the denominators of these two equal fractions are equal, if the lever be in equilibrium (art. 40). Hence the numerators are equal, or

Fig. 30.

$$P \times P\text{'s virtual velocity} = Q \times Q\text{'s virtual velocity.}$$

As a further and simpler example, take the case of the block and tackle described in article 41. Suppose the weight to be raised through a space h; then the virtual velocity of the weight is h, and is negative. Now as the distance between the block and tackle is less than before by the space h, and as the rope passes over this space n times, in order to keep the rope still tight the power will have to move through a space equal to nh. This is the virtual velocity of P, and is positive, and as $W = nP$, we see that

$$W \times W\text{'s virtual velocity} = P \times P\text{'s virtual velocity.}$$

47. **Friction.**—In the cases of the actions of machines which have been described, the resistances which are offered to motion have not been at all considered. The surfaces of bodies in contact are never perfectly smooth; even the smoothest present inequalities which can neither be detected by the touch nor by ordinary sight; hence when one body moves over the surface of another the elevations of one sink into the depressions of the other, like the teeth of wheels, and thus offer a certain resistance to motion; this is what is called *friction*. It must be regarded as a force which continually acts in opposition to actual or possible motion.

Friction is of two kinds: *sliding*, as when one body glides over another; this is least when the two surfaces in contact remain the same, as in the motion of an axle in its bearing; and *rolling* friction, which occurs when one body rolls over another, as in the case of an ordinary wheel. The latter is less than the former, for by the rolling the inequalities of one body are raised over those of the other.

Friction is directly proportional to the pressure of the two surfaces against each other. That portion of the pressure which is required to overcome friction is called the *coefficient of friction*.

Friction is independent of the extent of the surfaces in contact if the pressure is the same. Thus, suppose a board with a surface of a square decimetre resting on another board to be loaded with a weight of a kilogramme.

If this load be distributed over a similar board of two square decimetres surface, the total friction will be the same, while the friction per square centimetre is one half, for the pressure on each square centimetre is one half of what it was before. Friction is diminished by polishing and by smearing, but is increased by heat. It is greater as a body passes from the state of rest to that of motion than during motion, but seems independent of the velocity. The coefficient of friction depends on the nature of the substances in contact; thus for oak upon oak it is 0·418 when the fibres are parallel, and 0·293 when they cross; for beech upon beech it is 0·36. Greasy substances which are not absorbed by the body diminish friction; but increase it if they are absorbed. Thus moisture and oil increase, while tallow, soap, and graphite diminish, the friction of wooden surfaces. In the sliding friction of cast iron upon bronze the coefficient was found to be 0·25 without grease; with oil it was 0·17, fat 0·11, soap 0·03, and with a mixture of fat and graphite 0·02. The coefficient of rolling friction for cast-iron wheels on iron rails as in railways is about 0·004; for ordinary wheels on an ordinary road it is 0·04, hence a horse can draw ten times as great a load on rails as on an ordinary road.

As rolling friction is considerably less than sliding friction, it is a great saving of power to convert the latter into the former: as is done in the case of the casters of chairs and other furniture, and also in that of friction wheels. On the other hand, it is sometimes useful to change rolling into sliding friction, as when drags are placed on carriage wheels.

Without friction on the ground, neither men nor animals, neither ordinary carriages nor railway carriages, could move. Friction is necessary for the transmission of power from one wheel to another by means of bands or ropes; and without friction we could hold nothing in the hands.

48. Resistance to Motion in a Fluid Medium.—A body in moving through any medium such as air or water experiences a certain resistance; for the moving body sets in motion those parts of the medium with which it is in contact, whereby it loses an equivalent amount of its own motion.

This resistance increases with the surface of the moving body: thus a soap bubble or a snow flake falls more slowly than does a drop of water of the same weight. It also increases with the density of the medium; thus in rarefied air it is less than in air under the ordinary pressure; and in this again it is less than in water.

Fig. 31.

The influence of this resistance may be illustrated by means of the apparatus represented in fig. 31, which consists of two vanes, *w w*, fixed to a horizontal axis, *x x*, to which also is attached a bobbin *s*. The rotation of the vanes is effected by means of the falling of a weight attached to the string coiled round the bobbin. The vanes can be adjusted either at right angles or parallel to the axis. In the former position the vanes rotate rapidly when the weight is allowed to act; in the latter, however, where they press with

their entire surface against the air, the resistance greatly lessens the rapidity of rotation.

The resistance increases with the velocity of the moving body, and for moderate velocities is proportional to the square; for, supposing the velocities of a body made twice as great, it must displace twice as much matter, and must also impart to the displaced particles twice the velocity. For high velocities the resistance in a medium increases in a more rapid ratio than that of the square, for some of the medium is carried along with the moving body, and this, by its friction against the other portions of the medium, causes a loss of velocity.

It is this resistance which so greatly increases the difficulty and cost of attaining very high speeds in steam-vessels. Use is made, on the other hand, of this resistance in parachutes (fig. 151) and in the wind-vanes for diminishing the velocity of falling bodies (fig. 55), the principle of which is illustrated by the apparatus, fig. 31. Light bodies fall more slowly in air than heavy ones of the same surface, for the moving force is smaller compared with the resistance. The resistance to a falling body may ultimately equal its weight; it then moves uniformly forward with the velocity which it has acquired. Thus, a rain-drop falling from a height of 3,000 feet would, when near the ground, have a velocity of nearly 440 feet, or that of a musket-shot; owing, however, to the resistance of the air, its actual velocity is probably not more than 30 feet in a second. On railways the resistance of the air is appreciable; with a carriage exposing a surface of 22 square feet, it amounts to 16 or 17 pounds when the speed of the train is 16 feet a second or 11 miles an hour.

By observing the rate of diminution in the number of oscillations of a horizontal disc suspended by a thread, when immersed in water, Meyer determined the coefficient of the resistance of water, and found that at 10° it was equal to 0·01567 gramme on a square centimetre; and for air it was about $\frac{1}{16}$ as much.

49. **Uniformly Accelerated Rectilinear Motion.**—Let us suppose a body containing m units of mass to move from rest under the action of a force of F units, the body will move in the line of action of the force, and will acquire in each second an additional velocity f given by the equation

$$F = mf;$$

consequently, if v is its velocity at the end of t seconds, we have

$$v = ft. \tag{1}$$

To determine the space it will describe in t seconds, we may reason as follows:—The velocity at the time t being ft, that at a time $t + \tau$ will be $f(t + \tau)$. If the body moved uniformly during the time τ with the former velocity it would describe a space s equal to $ft\tau$; if with the latter velocity, a space s_1 equal to $f(t + \tau)\tau$. Consequently,

$$s_1 : s :: t + \tau : t;$$

therefore, when τ is indefinitely small, the limiting values of s and s_1 are equal. Now since the body's velocity is continually *increasing* during the time τ, the space actually described is greater than s, and less than s_1. But

since the limiting values of s and s_1 are equal, the limiting value of the space described is the same as that of s or s_1. In other words, if we suppose the whole time of the body's motion to be divided into any number of equal parts, if we determine the velocity of the body at the beginning of each of these parts, and if we ascertain the spaces described on the supposition that the body moves uniformly during each portion of time, the limiting value of the sum of these spaces will be the space actually described by the body.

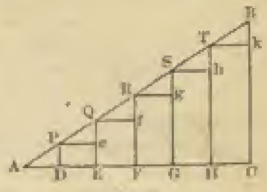

Fig. 32.

Draw a line AC (fig. 32) and at A construct an angle CAB, whose tangent equals f; divide AC into any number of equal parts in D, E, F,...and draw PD, QE, RF,... BC at right angles to AC, then since PD = AD $\times f$, QE = AE $\times f$, RF = AF $\times f$, BC = AC $\times f$, &c., PD will represent the velocity of the body at the end of the time represented by AD, and similarly QE, RF,...BC, will represent the velocity at the end of the times AE, AF,...AC. Complete the rectangles De, Ef, Fg... These rectangles represent the space described by the body on the above supposition during the second, third, fourth,...portions of the time. Consequently, the space actually described during the time AC is the limit of the sum of the rectangles; the limit being continually approached as the number of parts into which AC is divided is continually increased. But this limit is the area of the triangle ABC: that is $\frac{1}{2}$AC $\times$ CB or $\frac{1}{2}$AC $\times$ AC $\times f$. Therefore, if AC represents the time t during which the body describes a space s, we have

$$s = \tfrac{1}{2}ft^2. \tag{2}$$

Since this equation can be written

$$2fs = f^2t^2$$

we find, on comparison with equation (1), that

$$v^2 = 2fs. \tag{3}$$

To illustrate these equations, let us suppose the accelerative effect of the force to be 6; that is to say, that, in virtue of the action of the force, the body acquires in each successive second an additional velocity of 6 ft. per second, and let it be asked what, on the supposition of the body moving from rest, will be the velocity acquired and the space described at the end of 12 seconds; equations 1 and 2 enable us to answer that at that instant it will be moving at the rate of 72 ft. per second and will have described 432 ft.

The following important result follows from equation 2. At the end of the first, second, third, fourth, &c., second of the motion the body will have described $\frac{1}{2}f$, $\frac{1}{2}f \times 4$, $\frac{1}{2}f \times 9$, $\frac{1}{2}f \times 16$, &c., ft., and consequently *during* the first, second, third, fourth, &c., second of the motion will have described $\frac{1}{2}f$, $\frac{1}{2}f \times 3$, $\frac{1}{2}f \times 5$, $\frac{1}{2}f \times 7$, &c., ft., namely, spaces in arithmetical progression.

The results of the above article can be stated in the form of laws which apply to the state of a body moving from a state of rest under the action of a constant force:—

I. *The velocities are proportional to the times during which the motion has lasted.*

II. *The spaces described are proportional to the squares of the times employed in their description.*

III. *The spaces described are proportional to the squares of the velocities acquired during their description.*

IV. *The spaces described in equal successive periods of time increase by a constant quantity.*

Instead of supposing the body to begin to move from a state of rest, we may suppose it to have an initial velocity V, in the direction of the force. In this case equations 1, 2, and 3 can be easily shown to take the following forms, respectively :—

$$v = V + ft,$$
$$s = Vt + \tfrac{1}{2}ft^2,$$
$$v^2 = V^2 + 2fs.$$

If the body move in a direction opposite to that of the force, f must be reckoned negative.

The most important exemplification of the laws stated in the present article is in the case of a body falling freely *in vacuo.* Here the force causing the acceleration is that of gravity, and the acceleration produced is denoted by the letter g; it has already been stated (27 and 29) that the numerical value of g is 32·1912 at London, when the unit of time is a second and the unit of distance a foot. Adopting the metre as unit of distance the value of g at London is 9·8117.

50. **Motion on an Inclined Plane.**—Referring to (43), suppose the force P not to act; then the mass M is acted on by an unbalanced force Mg sin x, in the direction SR, consequently the accelerating force down the plane is g sin x, and the motion becomes a particular case of that discussed in the last article. If it begins to move from rest, it will at the end of t seconds acquire a velocity v given by the equation

$$v = gt \sin x,$$

and will describe a length s of the plane given by the equation

$$s = \tfrac{1}{2}gt^2 \sin x.$$

Also, if v is the velocity acquired while describing s feet of the plane,

$$v^2 = 2gs \sin x.$$

Hence (fig. 25) if a body slides down the plane from S to R the velocity which it acquires at R is equal to $\sqrt{2g \cdot \mathrm{RS} \sin \mathrm{R}}$ or $\sqrt{2g \cdot \mathrm{ST}}$; that is to say, the velocity which the body has at R does not depend on the angle x, but only on the perpendicular height ST. The same would be true if for RS we substituted any smooth curve, and hence we may state generally, that when a body moves along any smooth line under the action of gravity, the change of velocity it experiences in moving from one point to another is that due to the *vertical* height of the former point above the latter.

51. **Motion of Projectiles.**—The equations given in the above article apply to the case of a body thrown vertically upwards or downwards with a certain initial velocity. We will now consider the case of a heavy body

thrown in a horizontal direction. Let a, fig. 33, be such a body thrown with an initial velocity of v feet in a second, and let the line ab represent the space described in any interval; then, at the end of the 2, 3, 4...equal interval, the body, in virtue of its inertia, will have reached the points c d e, &c. But, during all this time the body is under the influence of gravity, which if it alone acted, would cause the body to fall through the distances represented on the vertical line; these are determined by the successive values of $\frac{1}{2}gt^2$, which is the formula for the space described by a freely falling body (49). The effect of the combined action of the two forces is that at the end of the first interval, &c., the body will be at b', at the end of the second interval at c', of the third at d', &c., the spaces bb', cc', dd'... being proportional to the squares of ab, ac, ad, respectively, and the line joining these points represents the path of the body. By taking the intervals of time sufficiently small we get a regularly curved line of the form known as the *parabola*.

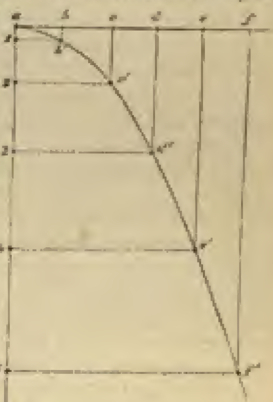

Fig. 33.

If the direction in which the body is thrown makes an angle of a with the horizon (fig. 34), then after t seconds it would have travelled a distance

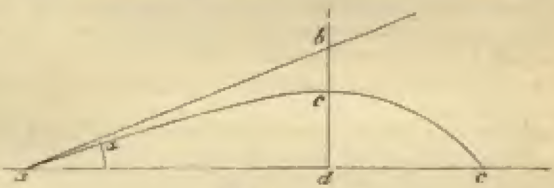

Fig. 34.

$ab = vt$, where v is the original velocity; during this time, however, it will have fallen through a distance $bc = \frac{1}{2}gt^2$; the height which it will have actually reached is $= bd - bc = vt \sin a - \frac{1}{2}gt^2$; and the horizontal distance will be $ad = ab \cos a = vt \cos a$. The *range* of the body, or the greatest distance through which it is thrown, will be reached when the height is again $= 0$; that is, when $vt \sin a - \frac{1}{2}gt^2 = 0$, from which $t = \frac{2v \sin a}{g}$. Introducing this value of t into the equation for the distance d, we have $d = \frac{2v^2 \sin a \cos a}{g}$, which by a trigonometrical transformation $= \frac{v^2 \sin 2a}{g}$. The greatest height is attained in half the time of flight, or when $t = \frac{v \sin a}{g}$, from which we get $h = \frac{v^2 \sin^2 a}{2g}$.

It follows from the formula that the height is greatest when $\sin a$ is

greatest, which is the case when it $= 90°$, or when the body is thrown vertically upwards; the range is greatest where $\sin 2a$ is a maximum, that is, when $2a = 90°$ or $a = 45°$.

In these formulæ it has been assumed that the air offers no resistance. This is, however, far from the case, and in practice, particularly if the velocity of projection is very great, the path differs from that of a parabola. Fig. 34 approximately represents the path, allowing for the resistance of the air. The divergence from the true theoretical path is the greater from the fact that in the modern rifled arms the projectiles are not spherical in shape, and also because, along with their motion of translation, they have, in consequence of the rifling, a rotatory motion about their axis.

52. **Composition of Velocities.**—The principle for the composition of velocities is the same as that for the composition of forces: this follows evidently from the fact that forces are measured by the momentum they communicate, and are therefore to one another in the same ratio as the velocities they communicate to the *same* body. Thus (fig. 6, art. 33) if the point has at any instant a velocity AB in the direction AP, and there is communicated to it a velocity AC in the direction AQ, it will move in the direction AR with a velocity represented by AD. And conversely, the velocity of a body represented by AD can be resolved into two component velocities AB and AC. This suggests the method of determining the motion of a body when acted on by a force in a direction transverse to the direction of its velocity; namely, suppose the time to be divided into a great number of intervals, and suppose the velocity actually communicated by the force to be communicated at once, then by the composition of velocities we can determine the motion during each interval, and therefore during the whole time; the actual motion is the limit to which the motion, thus determined, approaches when the number of intervals is increased.

53. **Motion in a Circle.—Centrifugal Force.**—When a body is once in motion, unless it be acted upon by some force, it will move uniformly forward in a straight line with unchanged velocity (26). If, therefore, a body moves uniformly in any other path than a straight line—in a circle, for instance—this must be because some force is constantly at work which continuously deviates it from this straight line.

We have already seen an example of this in the case of the motion of projectiles (51), and will now consider it in the case of central motion, or motion in a circle, of which we have an example in the motion of the celestial bodies or in the motion of a sling.

In the latter case, if the string is cut, the stone, ceasing to be acted upon by the tension of the string, will move in a straight line with the velocity which it already possesses; that is, in the direction of the tangent to the curve at the point where the stone was when the string was cut. The tension of the string, the effect of which is to pull the stone towards the centre of the circle, and to cause the stone to move in its circular path, is called the *centripetal* or central force; the reaction of the stone upon the string, which is equal and opposite to this force, is called its *centrifugal* force. The amount of these forces may be arrived at as follows:—

Let us suppose a body moving in a circle with given uniform velocity to be at the point a (fig. 35); then, had it not been acted on by a force in the

direction ac, it would, in a small succeeding interval of time t, have continued to move in the direction of the tangent at a, and have passed through a distance which we will represent by ab. In consequence, however, of this force it has not followed this direction, but has arrived at the point d on the curve; hence the force has made it traverse the distance $bd = ac$ in this interval. If f be the accelerating force which draws the body towards the centre, $ac = \frac{1}{2}ft^2$, and if ad be very small, it may be taken as equal to ab or vt, where v is the velocity of the moving body. Now if an is the diameter of the circle, the triangle adn is inscribed in a semicircle and is right-angled, whence $ad^2 = ac \times an = ac \times 2r$. Substituting their values for ad and ac in this equation, we find that $v^2t^2 = \frac{1}{2}ft^2 \times 2r$, from which $f = \frac{v^2}{r}$; that is, in order that a body, with a certain velocity, may move in a circle, it must be drawn to the centre by a force which is directly as the square of the velocity with which the body moves, and which is inversely as the radius of the circle. In order to express this in the ordinary units of weight, we must multiply the above expression by the mass, which gives $F = \frac{mv^2}{r}$ or $\frac{Wv^2}{gr}$. To keep the body in a circle an attraction towards the centre is needed, which is constantly equal to $\frac{mv^2}{r}$, and this attraction is constantly neutralised by the centrifugal force.

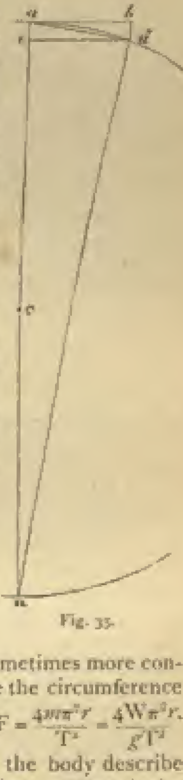

Fig. 35.

The above expression may be put in a form which is sometimes more convenient. If T be the time in seconds required to traverse the circumference $2\pi r$ with the velocity v, then $v^2 = \frac{4\pi^2 r^2}{T^2}$, from which $F = \frac{4m\pi^2 r}{T^2} = \frac{4W\pi^2 r}{gT^2}$.

If a rigid body rotates about a fixed axis, all parts of the body describe circumferences of various diameters, but all in the same time. The velocity of the motion of individual particles increases with the distance from the axis of rotation. By *angular velocity* is understood the velocity of a point at unit distance from the axis of rotation. If this is denoted by ω, the velocity v of a point at a distance from the axis is ωr, from which $\omega = \frac{v}{r} = \frac{2\pi}{T}$ and $f = r\omega^2$.

The existence of centrifugal force may be demonstrated by means of numerous experiments, such as the centrifugal railway. If a small can of water hung by the handle to a string be rapidly rotated in a vertical circle, no water will fall out, for, at a suitable velocity, the liquid will press against the bottom of the vessel with a force at right angles to the circle, and greater than its own weight.

54. **Motion in a Vertical Circle.**—Let ACBD be a circle whose plane is vertical and radius denoted by r. Suppose a point placed at A, and allowed to slide down the curve, what velocity will it have acquired on

reaching any given point P? Draw the vertical diameter CD, join CA, CP, and draw the horizontal lines AMB and PNP′. Now, assuming the curve to be smooth, the velocity acquired in falling from A to P is that due to MN, the vertical height of A above P (50); if, therefore, v denote the velocity of the point at P, we shall have

Fig. 36.

$$v^2 = 2g\text{MN}.$$

Now by similar triangles DCP, PCN we have

$$\text{DC} : \text{CP} :: \text{CP} : \text{CN};$$

consequently, if we denote by s the chord CP,

$$2r\text{NC} = s^2;$$

in like manner if a denote the chord CA,

$$2r\text{MC} = a^2,$$

therefore $$2r\,\text{MN} = a^2 - s^2,$$

and $$v^2 = \frac{g}{r}(a^2 - s^2).$$

Now v will have equal values when s has the same value, whether positive or negative, and for any one value of s there are two equal values of v, one positive and one negative. That is to say, since CP′ is equal to CP, the body will have the same velocity at P′ that it has at P, and at any point the body will have the same velocity whether it is going up the curve or down the curve. Of course it is included in this statement that if the body begins to move from A it will just ascend to a point B on the other side of C, such that A and B are in the same horizontal line. It will also be seen that at C the value of s is zero; consequently, if V is the velocity acquired by the body in falling from A to C, we have

$$\text{V} = a\sqrt{\frac{g}{r}};$$

and, on the other hand, if the body begins to move from C with a velocity V it will reach a point A such that the chord AC or a is given by the same equation. In other words, the velocity at the lowest point is proportional to the chord of the arc described.

Fig. 37.

55. **Motion of a Simple Pendulum.**—By a simple pendulum is meant a heavy particle suspended by a fine thread from a fixed point, about which it oscillates without friction. So far as its changes of velocity are concerned they will be the same as those of the point in the previous article; for the tension of the thread, acting at each position in a direction at right angles to that of the motion of the point, will no more affect its motion than the reaction of the smooth curve affects that of the point in the last article. The time of an oscillation—that is, the time in which the point moves from A to B—can be easily ascertained when the arc of vibration is small; that is, when the chord and the arc do not sensibly differ.

Thus, let AB (fig. 37) equal the arc or chord ACB (fig. 36); with centre C and radius AC or a describe a circle, and suppose a point to describe the circumference of that circle with a uniform velocity V or $a\sqrt{\frac{g}{r}}$. At any instant let the point be at Q, join CQ, draw the tangent QT, also draw QP at right angles and QN parallel to AB, then the angles NQT and CQP are equal. Now the velocity of Q resolved parallel to AB is V cos TQN or $a\sqrt{\frac{g}{r}}$ cos CQP; that is, if CP equals s, the velocity of Q parallel to AB is

$$\sqrt{\frac{g}{r}}\,\mathrm{PQ} \text{ or } \sqrt{\frac{g}{r}(a^2-s^2)}.$$

But if we suppose a point to move along AB in such a manner that its velocity in each position is the same as that of the oscillating body, its velocity at P would also equal $\sqrt{\frac{g}{r}(a^2-s^2)}$; and, therefore, this point would describe AB in the same time that Q describes the semicircumference AQB. If then be the required time of an oscillation, we have

$$t = \pi a \div a\sqrt{\frac{g}{r}} = \pi\sqrt{\frac{r}{g}}.$$

This result is independent of the length of the arc of vibration, provided its *amplitude*, that is AB, be small—not exceeding 4 or 5 degrees, for instance. It is evident from the formula that the time of a vibration is directly proportional to the square root of the length of the pendulum, and inversely proportional to the square root of the accelerating force of gravity.

As an example of the use of the formula we may take the following:—It has been found that 39·13983 inches is the length of a simple pendulum, whose time of oscillation at Greenwich is one second; the formula at once leads to an accurate determination of the accelerating force of gravity g; for using feet and seconds as our units we have $t = 1$, $r = 3{\cdot}26165$, and π stands for the known number 3·14159, therefore the formula gives us

$$g = (3{\cdot}14159)^2 \times 3{\cdot}26165 = 32{\cdot}1912.$$

This is the value employed in (29).

Other examples will be met with in the Appendix.

56. **Graphic Representation of the Changes of Velocity of an Oscillating Body.**—The changes which the velocity of a vibrating body undergoes may be graphically represented as follows:—Draw a line of indefinite length and mark off AH (fig. 38) to represent the time of one vibration, HH′ to represent the time of the second vibration, and so on. During the first vibration the velocity increases from zero to a maximum at the half-vibration, and then decreases during the second half-vibration from the maximum to zero. Consequently, a curved line or arc AQH may be drawn, whose ordinate QM at any point Q will represent the velocity of the body at the time represented

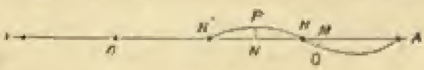

Fig. 38.

by AM. If a similar curved line or arc HPH′ be drawn, the ordinate PN of any point P will represent the velocity at a time denoted by AN. But since the *direction* of the velocity in the second oscillation is contrary to that of the velocity in the first oscillation, the ordinate NP must be drawn in the contrary direction to that of MQ. If, then, the curve be continued by a succession of equal arcs alternately on opposite sides of AD, the variations of the velocity of the vibrating body will be completely represented by the varying magnitudes of the ordinates of successive points of the curve. The last article shows this to be the curve of sines for a pendulum.

57. **Conical Pendulum.**—When a point P (fig. 39) is suspended from a point A as a simple pendulum, it can be caused to describe a horizontal circle with a uniform velocity V. A point moving in such a manner constitutes what is called a *conical pendulum*, and admits of many useful and interesting applications. We will, in this place, ascertain the relation which exists between the length r of the thread AP, the angle of the cone PAN or θ, and the velocity V. Since the point P moves in a circle whose radius is PN, with a velocity V, a force R must act on it in the direction PN given by the equation (53)

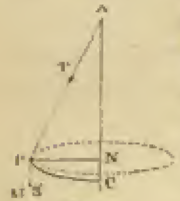

Fig. 39.

$$R = M\frac{V^2}{PN}.$$

Now the only forces acting are the tension of the thread T along PA, and the weight of the body Mg vertically ; consequently, their resultant must be a force R acting along PN. And therefore these forces will be parallel to the sides of the triangle ANP, so that (35)

$$R = Mg\frac{PN}{AN},$$

therefore

$$M\frac{V^2}{PN} = Mg\frac{PN}{AN},$$

or

$$V^2 = g\frac{PN^2}{AN}.$$

Now $$PN = r\sin\theta \text{ and } \frac{PN}{AN} = \tan\theta,$$

therefore

$$V^2 = gr\sin\theta\tan\theta.$$

One conclusion from this may be noticed. With centre A and radius AP, describe the arc PC. Now when the angle PAC is small, the sine, PN, does not sensibly differ from the chord, nor the cosine, AN, from the radius, therefore in this case we have

$$V^2 = g.\frac{(\text{chd PC})^2}{\text{radius}} \text{ or } V = \text{chd PC}\sqrt{\frac{g}{r}}.$$

On comparing this result with (54) we see that when the angle PAN is small, the velocity of P moving in a conical pendulum is the same as P

would have at the lowest point C if it oscillated as a simple pendulum; consequently, if we conceive the point P to be making small oscillations about the point A, and denote the velocity at the lowest point by V, and if, when at the extreme point of the arc of vibration, there is communicated to it a velocity V in a direction at right angles to the plane of vibration, its motion will be changed into that of a conical pendulum.

58. **Impulsive Forces.**—When a force acts on a body for an inappreciably short time, and yet sensibly changes its velocity, it is termed an *instantaneous* or *impulsive* force. Such a force is called into play when one body strikes against another. A force of this character is nothing but a finite though very large force, acting for a time so short that its duration is nearly, or quite, insensible. In fact, if M is the mass of the body, and the force contains Mf units, it will, in a time t, communicate a velocity ft; now, however small t may be, Mf and therefore f may be so large that ft may be of sensible or even considerable magnitude. Thus if M contain a pound of matter, and if the force contain ten thousand units, though t were so short as to be only the $\frac{1}{1000}$th of a second, the velocity communicated by the force would be one of 10 ft. per second. It is also to be remarked that the body will not sensibly move while this velocity is being communicated; thus, in the case supposed, the body would only move through $\frac{1}{2}ft^2$ or the $\frac{1}{200}$th of a foot while the force acts upon it.

When one body impinges on another it follows from the law of the equality of action and reaction (39) that whatever force the first body exerts upon the second, the second will exert an equal force upon the first in the opposite direction; now forces are proportional to the momenta generated in the same time; consequently, these forces generate, during the whole or any part of the time of impact, in the bodies respectively, equal momenta with contrary signs; and therefore the sum of the momenta of the two bodies will remain constant during and at the end of the impact. It is of course understood that if the two bodies move in contrary directions their momenta have opposite signs and the sum is an algebraical sum. In order to test the physical validity of this conclusion, Newton made a series of experiments, which may be briefly described thus:—Two balls A and B are hung from points C, D in the same horizontal line by threads in such a manner that their centres A and B are in the same horizontal line. With centre C and radius CA describe a semicircle EAF, and with centre D and radius DB describe a semicircle GBH on the wall in front of which the balls hang. Let A be moved back to R, and be allowed to descend to A; it there impinges on B; both A and B will now move, along the arch AF and BH respectively; let A and B come to their highest points at r and k respectively. Now if V denote the velocity with which A reaches the lowest point, v and u the velocities with which A and B leave the lowest points after impact, and r the radius AC, it follows from (54) that

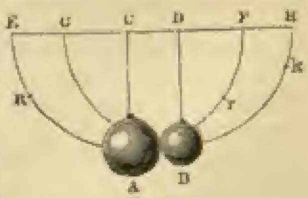

Fig. 40.

$$V = \text{chd } Ar\sqrt{\frac{g}{r}},\ v = \text{chd } Ar\sqrt{\frac{g}{r}},\ \text{and } u = \text{chd } Bk\sqrt{\frac{g}{r}};$$

therefore if A and B are the masses of the two balls, the momentum at the instant before impact was A × chd AR, and the momentum after impact was A × chd Ar + B × chd Bk. Now when the positions of the points R, r, and k had been properly corrected for the resistance of the air, it was found that these two expressions were equal to within quantities so small that they could be properly referred to errors of observation. The experiment succeeded equally under every modification, whether A impinged on B at rest or in motion, and whatever the materials of A and B might be.

59. **Direct Collision of Two Bodies.**—Let A and B be two bodies moving with velocities V and U respectively, along the same line, and let their mutual action take place in that line; if the one overtake the other, what will be their respective velocities at the instant after impact? We will answer this question in two extreme cases.

i. Let us suppose the bodies to be *quite inelastic*. In this case, when A touches B, it will continue to press against B until their velocities are equalised, when the mutual action ceases. For whatever deformation the bodies may have undergone, they have no tendency to recover their shapes. If, therefore, x is their common velocity after impact, we shall have $Ax + Bx$ their joint momentum at the end of impact, but their momentum before impact was AV + BU. Whence

$$(A + B)x = AV + BU,$$

an equation which determines x.

ii. Let us suppose the bodies *perfectly elastic*. In this case they recover their shapes, with a force exactly equal to that with which they were compressed. Consequently, the whole momentum lost by the one, and gained by the other, must be exactly double of that lost while compression took place; that is, up to the instant at which their velocities were equalised. But these are respectively $AV - Ax$ and $Bx - BU$; therefore, if v and u are the required final velocities,

$$Av = AV - 2(AV - Ax) \text{ or } v = -V + 2x$$
$$Bu = BU + 2(Bx - BU) \text{ or } u = 2x - U,$$

hence

$$(A + B)\,v = 2BU + (A - B)V$$

and

$$(A + B)\,u = 2AV - (A - B)U.$$

The following conclusion from these equations may be noticed: suppose a ball A, moving with a velocity V, to strike directly an equal ball B at rest. In this case A = B, and U = 0, consequently v = 0 and u = V: that is, the former ball A is brought to rest, and the latter B moves on with a velocity V. If now B strike on a third equal ball C at rest, B will in turn be brought to rest, and C will acquire the velocity V. And the same is true if there is a fourth, or fifth, or indeed any number of balls. This result may be shown with ivory balls, and if carefully performed is a very remarkable experiment.

60. **Work: Meaning of the Term.**—It has been pointed out (19, 26) that a moving body has no power of itself to change either the direction or the speed of its motion, and that, if any such change takes place, it is a proof that the body is acted upon by some external force. But although change of

motion thus always implies the action of force, forces are often exerted without causing any change in the motion of the bodies on which they act. For instance, when a ship is sailing at a uniform speed the force exerted on it by the wind causes no change in its motion, but simply prevents such a change being produced by the resistance of the water; or, when a railway-train is running with uniform velocity, the force of the engine does not change, but only maintains its motion in opposition to the forces, such as friction and the resistance of the air, which tend to destroy it.

These two classes of cases—namely, first, those in which forces cause a change of motion; and secondly, those in which they prevent, wholly or in part, such a change being produced by other forces—include all the effects to which the action of forces can give rise. When acting in either of these ways, a force is said to *do work*: an expression which is used scientifically in a sense somewhat more precise, but closely accordant with that in which it is used in common language. A little reflection will make it evident that, in all cases in which we are accustomed to speak of work being done—whether by men, horse-power, or steam-power, and however various the products may be in different cases—the physical part of the process consists solely in producing or changing motion, or in keeping up motion in opposition to resistance, or in a combination of these actions. The reader will easily convince himself of this by calling to mind what the definite actions are which constitute the work done by (say) a navvy, a joiner, a mechanic, a weaver; that done by a horse, whether employed in drawing a vehicle, or in turning a gin; or that of a steam-engine, whether it be used to drag a railway-train or to drive machinery. In all cases the work done is reducible, from a mechanical point of view, to the elements that have been mentioned, although it may be performed on different materials, with different tools, and with different degrees of skill.

It is, moreover, easy to see (comp. 52) that any possible change of motion may be represented as a gain by the moving body of an additional (positive or negative) velocity either in the direction of its previous motion, or at right angles to it; but a body which gains velocity is (27) said to be *accelerated*. Hence, what has been said above may be summed up as follows:—*When a force produces acceleration, or when it maintains motion unchanged in opposition to resistance, it is said to do* WORK.

61. **Measure of Work.**—In considering how work is to be measured, or how the relation between different quantities of work is to be expressed numerically, we have, in accordance with the above, to consider first, *work of acceleration*; and secondly, *work against resistance*. But in order to make the evaluation of the two kinds of work consistent, we must bear in mind that one and the same exertion of force will result in work of either kind according to the conditions under which it takes place: thus, the force of gravity acting on a weight let fall from the hand causes it to move with a continually accelerated velocity until it strikes the ground; but if the same weight, instead of being allowed to fall freely through the air, be hung to a cord passing round a cylinder by means of which various degrees of friction can be applied to hinder its descent, it can be made to fall with a very small and practically uniform velocity. Hence, speaking broadly, it may be said that, in the former case, the work done by gravity upon the weight is work of

acceleration only, while in the latter case it is work against resistance (friction) only. But it is very important to note that an essential condition, without which a force, however great, cannot do work either of one kind or the other, is that the thing acted on by it shall *move* while the force continues to act. This is obvious, for if no motion takes place it clearly cannot be either accelerated or maintained against resistance. The motion of the body on which a force acts being thus necessarily involved in our notion of work being done by the force, it naturally follows that, in estimating how much work is done, we should consider how much—that is to say, how far—the body moves while the force acts upon it. This agrees with the mode of estimating quantities of work in common life, as will be evident if we consider a very simple case—for instance, that of a labourer employed to carry bricks up to a scaffold: in such a case a double number of bricks carried would represent a double quantity of work done, but so also would a double height of the scaffold, for whatever amount of work is done in raising a certain number to a height of twenty feet, the same amount must be done again to raise them another twenty feet, or the amount of work done in raising the bricks forty feet is twice as great as that done when they are raised only twenty feet. It is also to be noted that no direct reference to *time* enters into the conception of a quantity of work: if we want to know how much work a labourer has done, we do not ask how long he has been at work, but what he has done—for instance, how many bricks he has carried, and to what height;—and our estimate of the total amount of work is the same whether the man has spent hours or days in doing it.

The foregoing relations between force and work may be put into definite mathematical language as follows:—If the point of application of a force moves in a straight line, and if the part of the force resolved along this line acts in the direction of the motion, the product of that component and the length of the line is the work done by the force. If the component acts in the opposite direction to the motion, the component may be considered as a resistance and the product is work done against the resistance. Thus, in (43), if we suppose a to move up the plane from R to S, the work done by P is $P \times RS$; the work done against the resistance W is $W \sin x \times RS$. It will be observed that if the forces are in equilibrium during the motion, so that the velocity of a is uniform, P equals $W \sin x$, and consequently the work done by the power equals that done against the resistance. Also since $RS \sin x$ equals ST, the work done against the resistance equals $W \times ST$. In other words, to raise W from R to S requires the same amount of work as to raise it from T to S.

If, however, the forces are not in equilibrium, the motion of a will not be uniform, but accelerated; the work done upon it will nevertheless still be represented by the product of the force into the distance through which it acts. In order to ascertain the relation between the amount of work done and the change produced by it in the velocity of the moving mass, we must recall one or two elementary mechanical principles. Let F be the resultant force resolved along the direction of motion, and S the distance through which its point of application moves: then, according to what has been said, the work done by the force $= FS$. Further, it has been pointed out (29) that a constant force is measured by the momentum produced by it in a unit of

time: hence, if T be the time during which the force acts, V the velocity of the mass M at the beginning of this period, and V_1 the velocity at the end, the momentum produced during the time T is $MV_1 - MV$, and consequently the momentum produced in a unit of time, or, in other words, the measure of the force, is

$$F = \frac{M(V_1 - V)}{T}.$$

The distance S through which the mass M moves while its velocity changes from the value V to the value V_1 is the same as if it had moved during the whole period T with a velocity equal to the average value of the varying velocity which it actually possesses. But a constant force acting upon a constant mass causes its velocity to change at a uniform rate; hence, in the present case, the average velocity is simply the arithmetical mean of the initial and final velocities, or

$$S = \tfrac{1}{2}(V_1 + V)T.$$

Combining this with the last equation, we get as the expression for the work done by the force F:

$$FS = \tfrac{1}{2}M(V_1^2 - V^2);$$

or, in words, *when a constant force acts on a mass so as to change its velocity, the work done by the force is equal to half the product of the mass into the change of the square of the velocity.*

The foregoing conclusion has been arrived at by supposing the force F to be constant, but it is easy to show that it holds good equally if F is the *average* magnitude of a force which varies from one part to another of the total distance through which it acts. To prove this, let the distance S be subdivided into a very great number n of very small parts each equal to s, so that $ns = S$. Then by supposing s to be sufficiently small, we may without any appreciable error consider the force as constant within each of these intervals and as changing suddenly as its point of application passes from one interval to the next. Let F_1, F_2, F_3 .... F_n, be the forces acting throughout the 1st, 2nd, 3rd ... nth interval respectively, and let the velocity at the end of the same intervals be v_1, v_2, v_3, ... v_n $(= V_1)$, respectively; then, for the work done in the successive intervals, we have—

$$F_1 s = \tfrac{1}{2}M(v_1^2 - V^2)$$
$$F_2 s = \tfrac{1}{2}M(v_2^2 - v_1^2)$$
$$F_3 s = \tfrac{1}{2}M(v_3^2 - v_2^2)$$
$$\cdot \qquad \cdot$$
$$\cdot \qquad \cdot$$
$$\cdot \qquad \cdot$$
$$F_n s = \tfrac{1}{2}M(v_n^2 - v_{n-1}^2) = \tfrac{1}{2}M(V_1^2 - v_{n-1}^2),$$

or, for the total work,

$$(F_1 + F_2 + F_3 + \ldots\ldots + F_n)s = \tfrac{1}{2}M(V_1^2 - v^2);$$

where the quantity of the left-hand side of the equation may also be written $\frac{F_1 + F_2 + \ldots + F_n}{n} ns = FS$, if we put F to stand for the average (or arithmetical mean) of the forces F_1, F_2, &c.

An important special case of the application of the above formula arises when either the initial or the final velocity of the mass M is nothing; that is to say, when the effect of the force is to make a body pass from a state of rest into one of motion, or from a state of motion into one of rest. The general expression then assumes one of the following forms, namely :—

$$FS = \tfrac{1}{2}MV_1^2 \text{ or,}$$
$$-FS = \tfrac{1}{2}MV^2;$$

the first of which denotes the quantity of work which must be done on a body of mass M in order to give to it the velocity V_1, while the second expresses the work that must be done in order to bring the same mass to rest when it is moving with the velocity V_0, the negative sign in the latter case showing that the force here acts *in opposition* to the actual motion, and is therefore to be regarded as a resistance.

In practice, the case which most frequently occurs is where work of acceleration and work against resistance are performed simultaneously. Thus, recurring to the inclined plane already referred to in art. 43; if the force P (where P is the constant force with which the string pulls W up the plane) be greater than W sin x, the body W will move up the incline with a continually increasing velocity, and if the point of application of P be displaced from R to S, the total amount of work done, namely, P × RS, consists of a portion = W sin x RS, done against the resistance of the weight W, and of a portion = (P – W sin x) RS expended in accelerating the weight. Hence, to determine the velocity v with which W arrives at the top of the incline we have the equation

$$(P - W \sin x)\ RS = \tfrac{1}{2}Wv^2;$$

for the portion of P which is in excess of what is required to produce equilibrium with the weight W, namely, P – W sin x, corresponds to the resultant force F supposed in the foregoing discussion, and RS to the distance through which this resultant force acts.

62. **Unit of Work.**—For strictly scientific purposes a unit of work is taken to be the work done by a unit of force when its point of application moves through one foot in the direction of its action; but, as a convenient and sufficiently accurate standard for practical purposes, the quantity of work which is done in lifting 1 pound through the height of 1 foot is commonly adopted as the unit, and this quantity of work is spoken of as one 'foot-pound.' It is, however, important to observe that the foot-pound is not perfectly invariable, since the weight of a pound, and therefore the work done in lifting it through a given height, differs at different places; being a little greater near the Poles than near the Equator.

On the metrical system the *kilogrammetre* is the unit; it is the weight of a kilogramme raised through a height of a metre. This is equal to 7·24 foot-pounds, and one foot-pound = ·1381 of a kilogrammetre.

63. **Energy.**—The fact that any agent is capable of doing work is usually expressed by saying that it possesses *Energy*, and the quantity of energy it possesses is measured by the amount of work it can do. For example, in the case of the inclined plane above referred to, the working power or energy of the force P is P × RS; and if this force acts under the conditions last supposed, by the time its own energy is exhausted (in consequence of its point of application having arrived at S, the limit of the range through which it is supposed able to act), it has conferred upon the weight W a quantity of energy equal to that which has been expended; for, in the first place, W has been raised through a vertical height equal to ST, and could by falling again through the same height do an amount of work represented by W × ST; and in the second place W can do work by virtue of the velocity that has been imparted to it, and can continue moving in opposition to any given resistance R through a distance s, such that

$$Rs = \tfrac{1}{2}Wv^2.$$

The energy possessed by the mass M in consequence of having been raised from the ground is commonly distinguished as *energy of position* or *potential energy*, and is measured by the product of the force tending to cause motion into the distance through which the point of application of the force is capable of being displaced in the direction in which the force acts. The energy possessed by a body in consequence of its velocity, is commonly distinguished as *energy of motion* or *kinetic energy*: it is measured by half the product of the moving mass into the square of its velocity.

64. **Varieties of Energy.**—It will be seen, on considering the definition of *work* given above, that a force is said to do work when it produces any change in the condition of bodies; for the only changes which, according to the definition of *force* given previously (26), a force is capable of producing, are changes in the state of rest or motion of bodies and changes of their place in opposition to resistances tending to prevent motion or to produce motion in an opposite direction. There are, however, many other kinds of physical changes which can be produced under appropriate conditions, and the recent progress of investigation has shown that the conditions under which changes of all kinds occur are so far analogous to those required for the production of work by mechanical forces that the term *work* has come to be used in a more extended sense than formerly, and is now often used to signify the production of any sort of physical change.

Thus work is said to be done when a body at a low temperature is raised to a higher temperature, just as much as when a weight is raised from a lower to a higher level; or again, work is done when any electrical, magnetic, or chemical change is produced. This extension of the meaning of the term *work* involves a similar extension of the meaning of *energy*, which in this wider sense may be defined as the *capacity for producing physical change*.

As examples of energy in this more general sense the following may be mentioned:—(*a*) the energy possessed by gunpowder in virtue of the mutual chemical affinities of its constituents, whereby it is capable of doing work by generating heat or by acting on a cannon-ball so as to change its state of rest into one of rapid motion; (*b*) the energy of a charged Leyden jar which, according to the way in which the jar is discharged, can give rise to changes

of temperature, to changes of chemical composition, to mechanical changes, or to changes of magnetic or electrical condition; (*e*) the energy of a red-hot ball which, amongst other effects it is capable of producing, can raise the temperature and increase the volume of bodies colder than itself, or can change ice into water or water into steam; the energy of the stretched string of a bow; here work has been consumed in stretching the string; when it is released the work reappears in the velocity imparted to the arrow.

65. **Transformations of Energy.**—It has been found by experiment that when one kind of energy disappears or is expended, energy of some other kind is produced, and that, under proper conditions, the disappearance of any one of the known kinds of energy can be made to give rise to a greater or less amount of any other kind. One of the simplest illustrations that can be given of this transformation of energy is afforded by the oscillations of a pendulum. When the pendulum is at rest in its lowest position it does not possess any energy, for it has no power of setting either itself or other bodies in motion or of producing in them any kind of change. In order to set the pendulum oscillating, work must be done upon it, and it thereafter possesses an amount of energy corresponding to the work that has been expended. When it has reached either end of its path, the pendulum is for an instant at rest, but it possesses energy by virtue of its position, and can do an amount of work while falling to its lowest position which is represented by the product of its weight into the vertical height through which its centre of gravity descends. When at the middle of its path the pendulum is passing through its position of equilibrium and has no power of doing work by falling lower; but it now possesses energy by virtue of the velocity which it has gained, and this energy is able to carry it up on the second side of its lowest position to a height equal to that from which it has descended on the first side. By the time it reaches this position the pendulum has lost all its velocity, but it has regained the power of falling: this, in its turn, is lost as the pendulum returns again to its lowest position, but at the same time it regains its previous velocity. Thus during every quarter of an oscillation, the energy of the pendulum changes from potential energy of position, into actual energy or energy of motion, or *vice versâ*.

A more complex case of the transformation of energy is afforded by a thermo-electric pile, the terminals of which are connected by a conducting wire: the application of energy in the form of heat to one face of the pile gives rise to an electric current in the wire, which, in its turn, reproduces heat, or by proper arrangements can be made to produce chemical, magnetic, or mechanical effects, such as those described below in the chapters on Electricity.

It has also been found that the transformations of energy always take place according to fixed proportions. For instance, when coal or any other combustible is burned, its chemical energy, or power of combining with oxygen, vanishes, and heat or thermal energy is produced, and the quantity of heat produced by the combustion of a given amount of coal is fixed and invariable. If the combustion take place under the boiler of a steam-engine, mechanical work can be obtained by the expenditure of part of the heat produced, and here again the quantitative relation between the heat expended and the work gained in place of it is perfectly constant.

66. **Conservation of Energy.**—Another result of great importance which has been arrived at by experiment is that the total amount of energy possessed by any system of bodies is unaltered by any transformations arising from the action of one part of the system upon another, and can only be increased or diminished by effects produced on the system by external agents. In this statement it is of course understood that in reckoning the sum of the energy of various kinds which the system may possess, those amounts of the different forms of energy which are mutually convertible into each other are taken as being numerically equal; or, what comes virtually to the same thing, the total energy of the system is supposed to be reduced—either actually, or by calculation from the known ratio of transformation of the various forms of energy—to energy of some one kind; then the statement is equivalent to this: that the total energy of any one form to which the energy of a given system of bodies is reducible is unalterable so long as the system is not acted on from without. Practically it is always possible, in one way or another, to convert the whole of the energy possessed by any body or system of bodies into heat, but it cannot be all converted without loss into any other form of energy; hence the principle stated at the beginning of this article can be enunciated in the closest conformity with the direct results of experiment, by saying that, so long as any system of bodies is not acted on from without, the total quantity of heat that can be obtained from it is unalterable by any changes which may go on within the system itself. For instance, a quantity of air compressed into the reservoir of an air-gun possesses energy which is represented partly by the heat which gives to it its actual temperature above the absolute zero (460), and partly by the work which the air can do in expanding. This latter portion can be converted into heat in various ways; as, for example, by allowing the air to escape through a system of capillary tubes, so fine that the air issues from them without any sensible velocity. If, however, the expanding air be employed to propel a bullet from the gun, it produces considerably less heat than in the case previously supposed, the deficiency being represented for a time by the energy of the moving bullet, but reappearing in the form of heat in the friction of the bullet against the air, and, when the motion of the bullet is destroyed, by striking against an inelastic obstacle at the same level as the gun. But whatever the mode and however numerous the intermediate steps by which the energy of the compressed air is converted into heat, the total quantity of heat finally obtainable from it is the same.

BOOK II.

GRAVITATION AND MOLECULAR ATTRACTION.

CHAPTER I.

GRAVITY. CENTRE OF GRAVITY. THE BALANCE.

67. **Universal Attraction; its Laws.**—*Universal attraction* is a force in virtue of which the material particles of all bodies tend incessantly to approach each other; it is a mutual action, however, which all bodies, at rest or in motion, exert upon one another, no matter how great or how small the space between them may be, or whether this space be occupied or unoccupied by other matter.

A vague hypothesis of the tendency of the matter of the earth and stars to a common centre was adopted even by Democritus and Epicurus. Kepler assumed the existence of a mutual attraction between the sun, the earth, and the other planets. Bacon, Galileo, and Hooke also recognised the existence of universal attraction. But Newton was the first who established the law, and the universality of gravitation.

Since Newton's time the attraction of matter by matter was experimentally established by Cavendish. This eminent English physicist succeeded by means of a delicate torsion balance (90) in rendering visible the attraction between a large leaden and a small copper ball.

The attraction between any two bodies is the resultant of the attractions of each molecule of the one upon every molecule of the other according to the law of Newton, which may be thus expressed: *the attraction between two material particles is directly proportional to the product of their masses and inversely proportional to the square of their distances asunder.* To illustrate this, we may take the case of two spheres which, owing to their symmetry, attract each other just as if their masses were concentrated in their centres. If without other alteration the mass of one sphere were doubled, tripled, &c., the attraction between them would be doubled, tripled, &c. If, however, the mass of one sphere being doubled, that of the other were increased three times, the distance between their centres remaining the same, the attraction would be increased six times. Lastly, if, without altering their masses, the distance between their centres were *increased* from 1 to 2, 3, 4, ... units, the attraction would be *diminished* to the 4th,

9th, 16th, ... part of its former intensity. In short, if we define the unit of attraction as that which would exist between two units of mass whose distance asunder was the unit of length, the attraction of two molecules, having the masses m and m', at the distance r, would be expressed by $\frac{m\,m'}{r^2}$.

68. **Terrestrial gravitation.**—The tendency of any body to fall towards the earth is due to the mutual attraction of that body and the earth, or to terrestrial gravitation, and is, in fact, merely a particular case of universal gravitation.

At any point of the earth's surface, the direction of gravity—that is, the line which a falling body describes—is called the *vertical* line. The vertical lines drawn at different points of the earth's surface converge very nearly to the earth's centre. For points situated on the same meridian the angle contained between the vertical lines equals the difference between the latitudes of those points.

The directions of the earth's attraction upon neighbouring bodies, or upon different molecules of one and the same body, must, therefore, be considered as parallel, for the two vertical lines form the sides of a triangle whose vertex is near the earth's centre, about 4,000 miles distant, and whose base is the small distance between the molecules under consideration.

A plane or line is said to be *horizontal* when it is perpendicular to the vertical line.

The vertical line at any point of the globe is generally determined by the *plumb-line* (fig. 41), which consists of a weight attached to the end of a string. It is evident that the weight cannot be in equilibrium, unless the direction of the earth's attraction upon it passes through the point of support, and therefore coincides with that of the string.

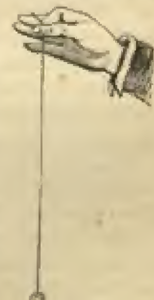

Fig. 41.

The horizontal plane is also determined with great ease, since it coincides, as will be afterwards shown, with the *level* surface of every liquid when in a state of equilibrium.

When the mean figure of the earth has been approximately determined, it becomes possible to compare the direction of the plumb-line at any place with that of the normal to the mean figure at that place. When any difference in these directions can be detected, it constitutes a *deviation* of the plumb-line, and is due to the attraction of some great mass of matter in the neighbourhood, such as a mountain. Thus, in the case of the mountain of Schehallien, in Perthshire, it was found by Dr. Maskelyne that the angle between the directions of two plumb-lines, one at a station to the north, and the other to the south, of the mountain, was greater by 11″6 than the angle between the normals of the mean surface of the earth at those points; in other words, each plumb-line was deflected by about 6″ towards the mountain. By calculating the volume and mass of the mountain, it was inferred from this observation that the mean density of the mountain was to that of the earth in the ratio of 5 : 9, and that the mean density of the earth is about five times that of water—a result agreeing

pretty closely with that deduced from Cavendish's experiments referred to in the last article.

69. **Centre of gravity, its experimental determination.**—Into whatever position a body may be turned with respect to the earth, there is a certain point, invariably situated with respect to the body, through which the resultant of the attracting forces between the earth and its several molecules always passes. This point is called the *centre of gravity*; it may be within or without the body, according to the form of the latter; its existence, however, is easily established by the following considerations: Let *m m′ m″*

Fig. 42.

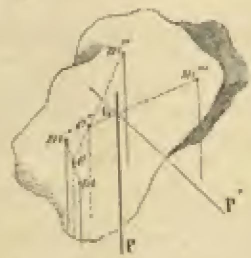

Fig. 43.

m‴. . . (fig. 42) be molecules of any body. The earth's attraction upon these molecules will constitute a system of parallel forces, having a common vertical direction, whose resultant, according to (36) will be found by seeking first the resultant of the forces which act on any two molecules, *m* and *m′*, then that of this resultant, and a third force acting on *m″*, and so on until we arrive at the final resultant, W, representing the weight of the body, and applied at a certain point, G. If the body be now turned into the position shown in fig. 43, the molecules *m*, *m′*, *m″*. . . will continue to be acted on by the same forces as before, the resultant of the forces on *m* and *m′* will still pass through the same point *o* in the line *mm′*, the following resultant will again pass through the same point *o′* in *om″*, and so on up to the final resultant P, which will still pass through the same point G, which is the *centre of gravity*.

To find the centre of gravity of a body is a purely geometrical problem; in many cases, however, it can be at once determined. For instance, the centre of gravity of a right line of uniform density is the point which bisects its length; in the circle and sphere it coincides with the geometrical centre; in cylindrical bars it is the middle point of the axis. The centre of gravity of a plane triangle is in the line which joins any vertex with the middle of the opposite side, and at a distance from the vertex equal to two-thirds of this line; in a cone or pyramid it is in the line which joins the vertex with the centre of gravity of the base, and at a distance from the vertex equal to three-fourths of this line. These rules, it must be remembered, presuppose that the several bodies are of uniform density.

In order to determine experimentally the centre of gravity of a body, it is suspended by a string in two different positions, as shown in figs. 44 and 45; the point where the directions AB and CD of the string in the two experiments intersect each other is the centre of gravity required. For the

resultant of the earth's attraction being a vertical force applied at the centre of gravity, the body can only be in equilibrium when this point lies vertically under the point of suspension; that is, in the prolongation of the suspended string. But the centre of gravity, being in AB as well as in CD, must coincide with the point of intersection of these two lines.

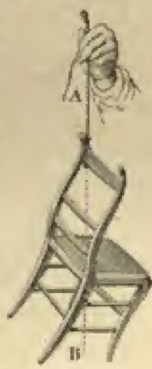

Fig. 44.

Fig. 45.

70. **Equilibrium of heavy bodies.**—Since the action of gravity upon a body reduces itself to a single vertical force applied at the centre of gravity and directed towards the earth's centre, equilibrium will be established only when this resultant is balanced by the resultant of other forces and resistances acting on the body at the fixed point through which it passes.

When only one point of the body is fixed, it will be in equilibrium if the vertical line through its centre of gravity passes through the fixed point. If more than one point is supported, the body will be in equilibrium if a vertical line through the centre of gravity passes through a point within the polygon formed by joining the points of support.

The Leaning Tower of Pisa continues to stand because the vertical line drawn through its centre of gravity passes within its base.

It is easier to stand on our feet than on stilts, because in the latter case the smallest motion is sufficient to cause the vertical line through the centre of gravity of our bodies to pass outside the supporting base, which is here reduced to a mere line joining the feet of the stilts. Again, it is impossible to stand on one leg if we keep one side of the foot and head close to a vertical wall, because the latter prevents us from throwing the body's centre of gravity vertically above the supporting base.

71. **Different states of equilibrium.**—Although a body supported by a fixed point is in equilibrium whenever its centre of gravity is in the vertical line through that point, the fact that the centre of gravity tends incessantly to occupy the lowest possible position leads us to distinguish between three states of equilibrium—*stable, unstable, neutral.*

A body is said to be in *stable equilibrium* if it tends to return to its first position after the equilibrium has been slightly disturbed. Every body is in this state when its position is such that the slightest alteration of the same elevates its centre of gravity; for the centre of gravity will descend again when permitted, and after a few oscillations the body will return to its original position.

The pendulum of a clock continually oscillates about its position of stable equilibrium, and an egg on a level table is in this state when its long axis is horizontal. We have another illustration in the toy represented in the adjoining fig. 46. A small figure cut in ivory is made to stand on one foot at the top of a pedestal by being loaded with two leaden balls, *a*, *b*, placed

sufficiently low to throw the centre of gravity, *g*, of the whole compound body below the foot of the figure. After being disturbed the little figure oscillates like a pendulum, having its point of suspension at the toe, and its centre of gravity at a lower point, *g*.

A body is said to be in *unstable equilibrium* when, after the slightest disturbance, it tends to depart still more from its original position. A body is in this state when its centre of gravity is vertically above the point of support, or higher than it would be in any adjacent

Fig. 46.

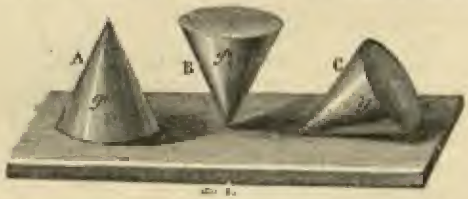

Fig. 47.

position of the body. An egg standing on its end, or a stick balanced upright on the finger, is in this state.

Lastly, if in any adjacent position a body still remains in equilibrium, its state of equilibrium is said to be *neutral*. In this case an alteration in the position of the body neither raises nor lowers its centre of gravity. A perfect sphere resting on a horizontal plane is in this state.

Fig. 47 represents three cones, A, B, C, placed respectively in stable, unstable, and neutral equilibrium upon a horizontal plane. The letter *g* in each shows the position of the centre of gravity.

72. **The balance.**—The balance is an instrument for determining the relative weights or masses of bodies. There are many varieties.

The ordinary balance (fig. 48) consists of a lever of the first kind, called the *beam*, AB, with its fulcrum in the middle; at the extremities of the beam are suspended two scale pans, C and D, one intended to receive the object to be weighed, and the other the counterpoise. The fulcrum consists of a steel prism, *n*, commonly called a *knife edge*, which passes through the beam, and rests with its sharp edge, or *axis of suspension*, upon two supports; these are formed of agate, in order to diminish the friction. A needle or pointer is fixed to the beam, and oscillates with it in front of the graduated arc, *a*; when the beam is perfectly horizontal the needle points to the zero of the graduated arc.

Since by (40) two equal forces in a lever of the first kind cannot be in equilibrium unless their leverages are equal, the length of the arms *n*A and *n*B ought to remain equal during the process of weighing. To secure this the scales are suspended from hooks, whose curved parts have sharp edges, and rest on similar edges at the ends of the beam. In this manner the scales are in effect supported on mere points, which remain unmoved during the oscillations of the beam. This mode of suspension is represented in fig. 48.

73. **Conditions to be satisfied by a balance.**—A good balance ought to satisfy the following conditions :—

i. *The two arms of the beam ought to be precisely equal*, otherwise, according to the principle of the lever, unequal weights will be required to produce equilibrium. To test whether the arms of the beam are equal, weights are placed in the two scales until the beam becomes horizontal; the contents of the scales being then interchanged, the beam will remain

Fig. 48.

horizontal if its arms are equal, but if not, it will descend on the side of the longer arm.

ii. *The balance ought to be in equilibrium when the scales are empty*, for otherwise unequal weights must be placed in the scales in order to produce equilibrium. It must be borne in mind, however, that the arms are not necessarily equal, even if the beam remains horizontal when the scales are empty; for this result might also be produced by giving to the longer arm the lighter scale.

iii. *The beam being horizontal, its centre of gravity ought to be in the same*

vertical line with the edge of the fulcrum, and a little below the latter, for otherwise the beam would not be in stable equilibrium (71).

The effect of changing the position of the centre of gravity may be shown by means of a beam (fig. 49), whose fulcrum being the nut of a screw, *a*, can be raised or lowered by turning the screw-head, *b*.

When the fulcrum is at the top of the groove *c*, in which it slides, the centre of gravity of the beam is below its edge, and the latter oscillates freely

Fig. 49.

about a position of stable equilibrium. By gradually lowering the fulcrum its edge may be made to pass through the centre of gravity of the beam when the latter is in neutral equilibrium; that is to say, it no longer oscillates, but remains in equilibrium in all positions. When the fulcrum is lowered still more, the centre of gravity passes above its edge, the beam is in a state of unstable equilibrium, and is overturned by the least displacement.

74. **Delicacy of the balance.**—A balance is said to be *delicate* when a very small difference between the weights in the scales causes a perceptible deflection of the pointer.

Let A and B (figs. 50 and 51) be the points from which the scale pans are suspended, and C the axis of suspension of the beam. A, B, and C are

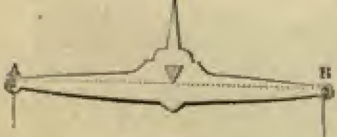

Fig. 50.

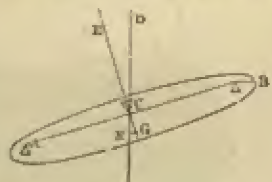

Fig. 51.

supposed to be in the same straight line, according to the usual arrangement. Suppose weights P and Q to be in the pans, suspended from A and B respectively, and let G be the centre of gravity of the beam; then the beam will come to rest in the position shown in the figure, where the line DCN is vertical, and ECG is the direction of the pointer. According to the above statement, the greater the angle ECD for a given difference between P and Q, the greater is the delicacy of the balance. Draw GN at right angles to CG.

Let W be the weight of the beam, then from the properties of the lever it follows that measuring moments with respect to C, the moment of P equals the sum of the moments of Q and W, a condition which at once leads to the relation

$$(P - Q)\ AC = W \times GN$$

Now it is clear that for a given value of CG the angle GCN (that is, ECD, which measures the delicacy) is great as GN is greater: and from the formula it is clear that for a given value of $P-Q$ we shall have GN greater as AC is greater, and as W is less. Again, for a given value of GN the angle GCN is greater as CG is less. Hence the means of rendering a balance delicate are:—

i. *To make the arms of the balance long.*

ii. *To make the weight of the beam as small as is consistent with its rigidity.*

iii. *To bring the centre of gravity of the beam a very little below the point of support.*

Moreover, since friction will always oppose the action of the force that tends to preponderate, the balance will be rendered more delicate by diminish-

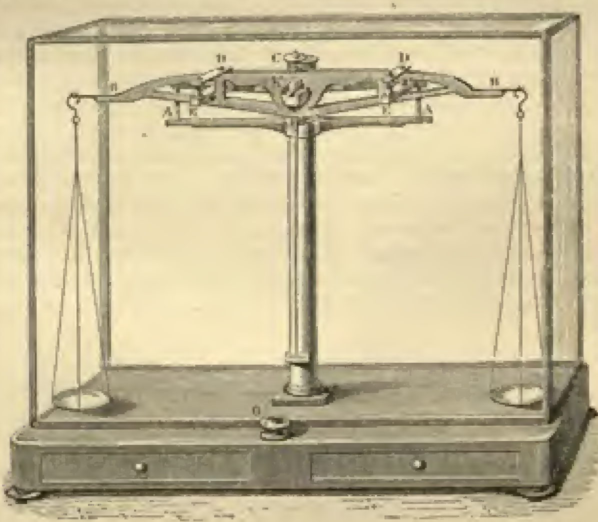

Fig. 52.

ing friction. To secure this advantage the edges from which the beam and scales are suspended are made as sharp and as hard as possible, and the supports on which they rest are very smooth and hard. This is effected by the use of agate knife edges. And, further, the pointer is made long, since its elongation renders a given deflection more perceptible by increasing the arc which its end describes.

75. **Physical and chemical balances.**—Fig. 52 represents one of the accurate balances ordinarily used for chemical analysis. Its sensitiveness is such that when charged with a kilogramme (1,000 grms.) in each scale an excess of a milligramme ($\frac{1}{1000}$th of a grm.) in either scale produces a very perceptible deflection of the index.

In order to protect the balance from air currents, dust, and moisture, it is always, even when weighing, surrounded by a glass case, whose front

slides up and down, to enable the operator to introduce the objects to be weighed. Where extreme accuracy is desired the case is constructed so that the space may be exhausted and the weighing made *in vacuo*.

In order to preserve the edge of the fulcrum as much as possible, the whole beam, BB, with its fulcrum K, can be raised from the support on which the latter rests by simply turning the button O outside the case.

The horizontality of the beam is determined by means of a long index, which points downwards to a graduated arc near the foot of the supporting pillar. Lastly, the button C serves to alter the sensitiveness of the balance; by turning it, the centre of gravity of the beam can be made to approach or recede from the fulcrum (73).

76. **Method of double weighing.**—Even if a balance be not perfectly accurate, the true weight of a body may still be determined by its means. To do so, the body to be weighed is placed in one scale, and shot or sand poured into the other until equilibrium is produced; the body is then replaced by known weights until equilibrium is re-established. The sum of these weights will necessarily be equal to the weight of the body, for, acting under precisely the same circumstances, both have produced precisely the same effect.

The exact weight of a body may also be determined by placing it successively in the two pans of a balance, and then deducing its true weight.

For, having placed in one pan the body to be weighed, whose true weight is x, and in the other the weight p, required to balance it, let a and b be the arms of levers corresponding to x and p. Then from the principle of the lever (40) we have $ax = pb$. Similarly if p_1 is the weight when the body is placed in the other pan, then $bx = ap_1$. Hence $abx^2 = abpp_1$, from which $x = \sqrt{pp_1}$.

CHAPTER II.

LAWS OF FALLING BODIES. INTENSITY OF TERRESTRIAL GRAVITY. THE PENDULUM.

77. **Laws of falling bodies.**—Since a body falls to the ground in consequence of the earth's attraction on *each* of its molecules, it follows that everything else being the same, all bodies, great and small, light and heavy, ought to fall with equal rapidity, and a lump of sand without cohesion should, during its fall, retain its original form as perfectly as if it were compact stone. The fact that a stone falls more rapidly than a feather is due solely to the unequal resistances opposed by the air to the descent of these bodies; *in a vacuum all bodies fall with equal rapidity.* To demonstrate this by experiment a glass tube about two yards long (fig. 53) may be taken, having one of its ends completely closed, and a brass cock fixed to the other. After having introduced bodies of different weights and densities (pieces of lead, paper, feather, &c.) into the tube, the air is withdrawn from it by an air-pump, and the cock closed. If the tube be now suddenly reversed, all the bodies will fall equally quickly. On introducing a little air and again inverting the tube, the lighter bodies become slightly retarded, and this retardation increases with the quantity of air introduced.

Fig. 53.

The resistance opposed by the air to falling bodies is especially remarkable in the case of liquids. The Staubbach in Switzerland is a good illustration; an immense mass of water is seen falling over a high precipice, but before reaching the bottom it is shattered by the air into the finest mist. In a vacuum, however, liquids fall like solids without separation of their molecules. The *water-hammer* illustrates this: the instrument consists of a thick glass tube about a foot long, half filled with water, the air having been expelled by ebullition previous to closing one extremity with the blow-pipe. When such a tube is suddenly inverted, the water falls in one undivided mass against the other extremity of the tube, and produces a sharp dry sound, resembling that which accompanies the shock of two solid bodies.

From Newton's law (67) it follows that when a body falls to the earth the force of attraction which causes it to do so increases as the body approaches the earth. Unless the height from which the body falls, however, be very great, this increase will be altogether inappreciable, and the force in question may be considered as constant and continuous. If the resistance of the air were removed, therefore, the motion of all bodies falling to the earth would be uniformly accelerated, and would obey the laws already explained (49).

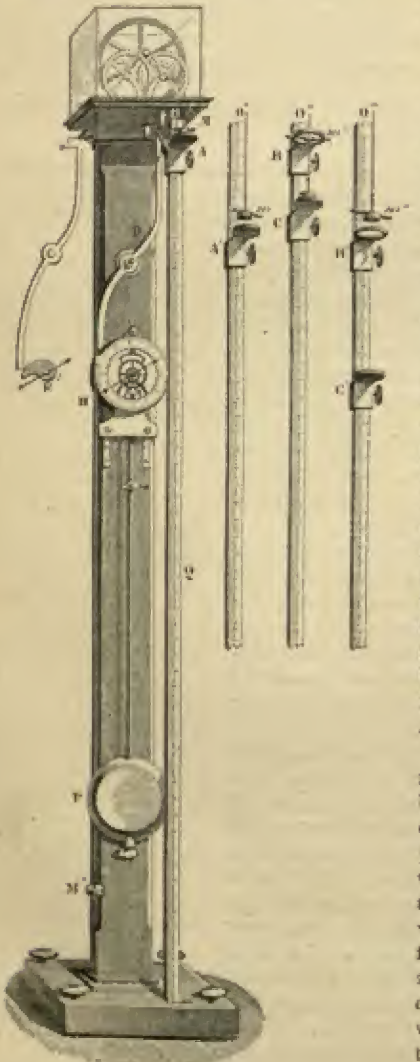

Fig. 54.

78. **Atwood's machine.**—Several instruments have been invented for illustrating and experimentally verifying the laws of falling bodies. Galileo, who discovered these laws in the early part of the seventeenth century, illustrated them by means of bodies falling down inclined planes. The great object of all such instruments is to diminish the rapidity of the fall of bodies without altering the character of their motion, for by this means their motion may not only be better observed, but it will be less modified by the resistance of the air (48).

The most convenient instrument of this kind is that invented by Atwood at the end of the last century, and represented in fig. 54. It consists of a stout pillar of wood, about $2\frac{1}{2}$ yards high, at the top of which is a brass pulley, whose axle rests and turns upon four other wheels, called *friction wheels*, inasmuch as they serve to diminish friction. Two equal weights, M and M′, are attached to the extremities of a fine silk thread, which passes round the pulley; a time-piece, H, fixed to the pillar, is regulated by a seconds pendulum, P, in the usual way; that is to say, the oscillations of the pendulum are communicated to a ratchet,

whose two teeth, as seen in the figure, fit into those of the ratchet wheel. The axle of this wheel gives motion to the seconds hand of the dial, and also to an eccentric behind the dial, as shown at E by a separate figure. This eccentric plays against the extremity of a lever D, which it pushes until the latter no longer supports the small plate, *i*, and thus the weight M, which at first rested on this plate, is suddenly exposed to the free action of gravity. The eccentric is so constructed that the little plate *i* falls precisely when the hand of the dial points to zero.

The weights M and M′, being equal, hold each other in equilibrium; the weight M, however, is made to descend slowly by putting a small bar or overweight *m* upon it; and to measure the spaces which it describes, the rod or scale, Q, is divided into feet and inches, commencing from the plate *i*. To complete the instrument, there are a number of plates, A, A′, C, C′, and a number of rings, B, B′, which may be fixed by screws at any part of the scale. The plates arrest the descending weight M, the rings only arrest the bar or overweight *m*, which was the cause of motion, so that after passing through them, the weight M, in consequence of its inertia, will move on uniformly with the velocity it had acquired on reaching the ring. The several parts of the apparatus being described, a few words will suffice to explain the method of experimenting.

Let the hand of the dial be placed behind the zero point, the lever D adjusted to support the plate *i*, on which the weight M with its overweight *m* rests, and the pendulum put in motion. As soon as the hand of the dial points to zero the plate *i* will fall, the weights M and *m* will descend, and by a little attention and a few trials it will be easy to place a plate A so that M may reach it exactly as the dial indicates the expiration of one second. To make a second experiment, let the weights M and *m*, the plate *i*, and the lever D, be placed as at first; remove the plate A, and in its place put a ring, B, so as to arrest the overweight *m* just when the weight M would have reached A; on putting the pendulum in motion again it will be easy, after a few trials, to put a plate, C, so that the weight M may fall upon it precisely when the hands of the dial point to two seconds. Since the overweight *m* in this experiment was arrested by the ring B at the expiration of one second, the space BC was described by M in one second purely in virtue of its own inertia, and consequently by (25) BC will indicate the velocity of the falling mass at the expiration of one second.

Proceeding in the same manner as before, let a third experiment be made in order to ascertain the point B′ at which the weights M and *m* arrive after the lapse of two seconds, and putting a ring at B′, ascertain by a fourth experiment the point C′ at which M arrives alone, three seconds after the descent commenced; B′C′ will then express the velocity acquired after a descent of two seconds. In a similar manner, by a fifth and sixth experiment, we may determine the space OB″ described in three seconds, and the velocity B″C″ acquired during those three seconds, and so on; we shall find that B′C′ is twice, and B″C″ three times as great as BC—in other words, that the velocities BC, B′C′, B″C″, increase in the same proportion as the times (1, 2, 3, . . . seconds) employed in their acquirement. By the definition (49), therefore, the motion is uniformly accelerated. The same experiments will also serve to verify and illustrate the four laws of uniformly

accelerated motion as enunciated in (49). For example, the spaces OB, OB′, OB″, ... described from a state of rest in 1, 2, 3, ... seconds will be found to be proportional to the numbers 1, 4, 9; . . . that is to say, to the squares of those numbers of seconds, as stated in the third law.

Lastly, if the overweight *m* be changed, the acceleration or velocity BC acquired per second will also be changed, and we may easily verify the assertion in (29), that force is proportional to the product of the mass moved into the acceleration produced in a given time. For instance, assuming the pulley to be so light that its inertia can be neglected, if *m* weighed half an ounce, and M and M′ each $15\frac{3}{4}$ ounces, the acceleration BC would be found to be six inches; whilst if *m* weighed 1 ounce, and M and M′ each $63\frac{1}{2}$ ounces, the acceleration BC would be found to be three inches.

Now in these cases the forces producing motion, that is the overweights, are in the ratio of 1 : 2; while the products of the masses and the accelerations are in the ratio of $(\frac{1}{2} + 15\frac{3}{4} + 15\frac{3}{4}) \times 6$ to $(1 + 63\frac{1}{2} + 63\frac{1}{2}) \times 3$; that is, they are also in the ratio of 1 : 2. Now the same result is obtained in whatever way the magnitudes of *m*, M, and M′ are varied, and consequently in all cases the ratio of the forces producing motion equals the ratio of the momenta generated.

79. **Morin's apparatus.**—The principle of this apparatus, the original idea of which is due to General Poncelet, is to make the body in falling trace its own path. Figure 55 gives a view of the whole apparatus, and figure 56 gives the details. The apparatus consists of a wooden framework, about 7 feet high, which holds in a vertical position a very light wooden cylinder, M, which can turn freely about its axis. This cylinder is coated with paper divided into squares by equidistant horizontal and vertical lines. The latter measure the path traversed by the body falling along the cylinder, while the horizontal lines are intended to divide the duration of the fall into equal parts.

The falling body is a mass of iron, P, provided with a pencil which is pressed against the paper by a small spring. The iron is guided in its fall by two light iron wires which pass through guide-holes on the two sides. The top of this mass is provided with a tipper which catches against the end of a bent lever, AC. This being pulled by the string K attached at A, the weight falls. If the cylinder M were fixed, the pencil would trace a straight line on it; but if the cylinder moves uniformly, the pencil traces the line *mn*, which serves to deduce the law of the fall.

The cylinder is rotated by means of a weight, Q, suspended to a cord which passes round the axle G. At the end of this is a toothed wheel, *c*, which turns two endless screws, *a* and *b*, one of which turns the cylinder, and the other two vanes, *x* and *x′* (fig. 56). At the other end is a ratchet wheel, in which fits the end of a lever, B; by pulling at a cord fixed to the other end of B, the wheel is liberated, the weight Q descends, and the whole system begins to turn. The motion is at first accelerated, but as the air offers a resistance to the vanes (48), which increases as the rotation becomes more rapid, the resistance finally equals the acceleration which gravity tends to impart. From this time the motion becomes uniform. This is the case when the weight Q has traversed about three-quarters its course; at this moment the weight P is detached by pulling the cord K, and the pencil then traces the curve *mn*.

If, by means of this curve, we examine the double motion of the pencil on the small squares which divide the paper, we see that, for displacements 1, 2, 3, ... in a horizontal direction, the displacements are 1, 4, 9 . . . . in a vertical direction. This shows that the paths traversed in the direction of the fall are directly as the squares of the lines in the direction of the rotation, which verifies the second law of falling bodies.

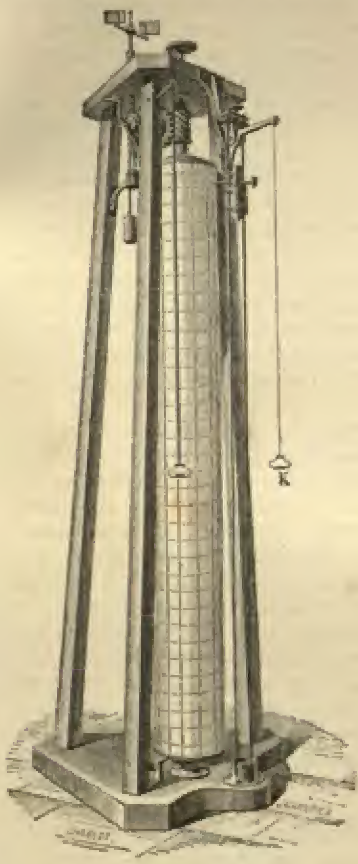

Fig. 55.

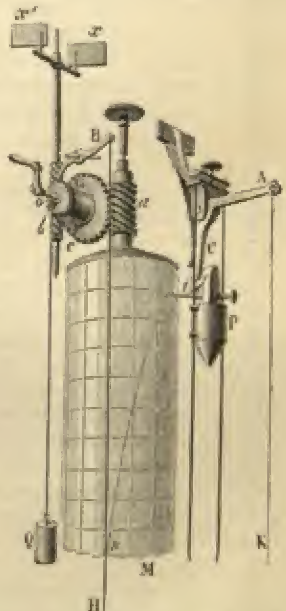

Fig. 56.

From the relation which exists between the two dimensions of the curve *mn*, it is concluded that this curve is a *parabola*.

80. **The length of the compound pendulum.**—The formula deduced in article (55) and the conclusions which follow therefrom refer to the case of the simple or mathematical pendulum; that is, to a single heavy point suspended by a thread without weight. Such a pendulum has only an imaginary

existence, and any pendulum which does not realise these conditions is called a *compound* or *physical* pendulum. The laws for the time of vibration of a compound pendulum are the same as those which regulate the motion of the simple pendulum, though it will be necessary to define accurately what is meant by the *length* of such a pendulum. A compound pendulum being formed of a heavy rod terminated by a greater or less mass, it follows that the several material points of the whole system will strive to perform their oscillations in different times, their distances from the axis of suspension being different, and the more distant points requiring a longer time to complete an oscillation. From this, and from the fact that being points of the same body they must all oscillate together, it follows that the motion of the points near the axis of suspension will be retarded, whilst that of the more distant points will be accelerated, and between the two extremities there will necessarily be a series of points whose motion will be neither accelerated nor retarded, but which will oscillate precisely as if they were perfectly free and unconnected with the other points of the system. These points, being equidistant from the axis of suspension, constitute a parallel axis known as the *axis of oscillation*; and it is to the distance between these two axes that the term *length of the compound pendulum* is applied: we may say, therefore, that *the length of a compound pendulum is that of the simple pendulum which would describe its oscillations in the same time.*

Fig. 57

Huyghens, the celebrated Dutch physicist, discovered that the axes of suspension and oscillation are mutually convertible; that is to say, the time of oscillation will remain unaltered when the pendulum is suspended from its axis of oscillation. This enables us to determine experimentally the length of the compound pendulum. For this purpose the *reversible pendulum* devised by Bohnenberger and Kater may be used. One form of this (fig. 57) is a rod with the knife-edges *a* and *b* turned towards each other. W and V are lens-shaped masses the relative positions of which may be varied. By a series of trials a position can be found such that the number of oscillations of the pendulum in a given time is the same whether it oscillates about the axis *a* or the axis *b*. This being so, the distance *ab* represents the length *l* of a simple pendulum which has the same time of oscillation. From the value of *l*, thus obtained, it is easy to determine the length of the seconds pendulum.

The length of the *seconds* pendulum—that is to say, of the pendulum which makes one oscillation in a second—varies, of course, with the intensity of gravity. The following table gives its value at the sea level at various places. The accelerative effect of gravity at these places, according to formula (55), is obtained in feet and metres, by multiplying the length of the seconds pendulum, reduced to feet and metres respectively, by the square of 3·14159.

	Latitude.	Length of Pendulum in inches.	Acceleration of Gravity in feet.	metres.
Hammerfest . .	70° 40′ N.	39·1948	32·2364	9·8258
Manchester . .	53 29	39·1472	32·1972	9·8132
Konigsberg . .	54 42	39·1507	32·2002	9·8142
Berlin . . .	52 30	39·1439	32·1945	9·8124
Greenwich . .	51 29	39·1398	32·1912	9·8115
Paris . . .	48 50	39·1285	32·1819	9·8039
New York . .	40 43	39·1012	32·1594	9·8019
Washington . .	38 54	39·0968	32·1558	9·8006
Madras . . .	13 4	39·0268	32·0992	9·7836
Ascension . .	7 56	39·0242	32·0939	9·7817
St. Thomas . .	0 25	39·0207	32·0957	9·7826
Cape of Good Hope	33 55 S.	39·0780	32·1404	9·7962

Consequently, $\frac{1}{2}g$ or the space described in the first second of its motion by a body falling *in vacuo* from a state of rest (49) is

16·0478 feet or 4·891 metres at St. Thomas,
16·0956 „ „ 4·905 „ at London, and
16·1182 „ „ 4·913 „ at Hammerfest.

In all calculations, which are used for the sake of illustration, we may take 32 feet or 9·8 metres as the accelerative effect due to gravity.

From observations of this kind, after applying the necessary corrections, and taking into account the effect of rotation (83), the form of the earth can be deduced.

81. **Verification of the laws of the pendulum.**—In order to verify the laws of the simple pendulum (55) we are compelled to employ a compound one, whose construction differs as little as possible from that of the former. For this purpose a small sphere of a very dense substance, such as lead or platinum, is suspended from a fixed point by means of a very fine metal wire. A pendulum thus formed oscillates almost like a simple pendulum, whose length is equal to the distance of the centre of the sphere from the point of suspension.

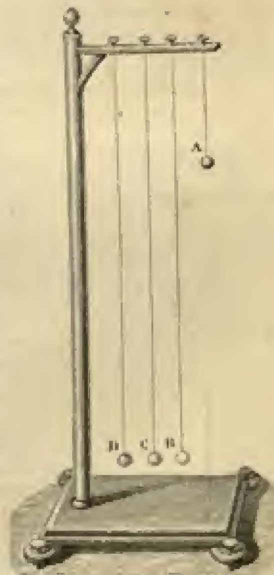

Fig. 58.

In order to verify the isochronism of small oscillations, it is merely necessary to count the number of oscillations made in equal times, as the amplitudes of these oscillations diminish from 3 degrees to a fraction of a degree; this number is found to be constant.

That the time of vibration is proportional to the square root of the length is verified by causing pendulums, whose lengths are as the numbers 1, 4, 9, ... to oscillate simultaneously. The corresponding numbers of oscillations in a given

time are then found to be proportional to the fractions, 1, $\frac{1}{2}$, $\frac{1}{3}$, &c., ... which shows that the times of oscillation increase as the numbers 1, 2, 3, ... &c.

By taking several pendulums of exactly equal length, B, C, D (fig. 58), but with spheres of different substances—lead, copper, ivory—it is found that, neglecting the resistance of the air, these pendulums oscillate in equal times, thereby showing that the accelerative effect of gravity on all bodies is the same at the same place.

By means of an arrangement resembling the above, Newton verified the fact that the *masses* of bodies are determined by the balance; which, it will be remarked, lies at the foundation of the measure of force (29). For it will be seen on comparing (54) and (55) with (50) that the law of the time of a small oscillation is obtained on the supposition that the force of gravity on all bodies is represented by Mg, in which M is determined by the balance. In order to verify this, he had made two round equal wooden boxes; he filled one with wood, and as nearly as possible in the centre of oscillation of the other he placed an equal weight of gold. He then suspended the boxes by threads eleven feet long, so that they formed pendulums exactly equal so far as weight, figure, and resistance of the air were concerned. Their oscillations were performed in exactly the same time. The same results were obtained when other substances were used, such as silver, lead, glass, sand, salt, wood, water, corn. Now all these bodies had equal weights, and if the inference, that therefore they had equal masses, had been erroneous, by so much as the one-thousandth part of the whole, the experiment would have detected it.

82. **Application of the pendulum to clocks.**—The regulation of the motion of clocks is effected by means of pendulums, that of watches by balance-springs. Pendulums were first applied to this purpose by Huyghens in 1658, and in the same year Hooke applied a spiral spring to the balance of a watch. The manner of employing the pendulum is shown in fig. 59. The pendulum rod passing between the prongs of a fork a communicates its motion to a rod b, which oscillates on a horizontal axis o. To this axis is fixed a piece mn called an *escapement* or *crutch*, terminated by two projections or *pallets*, which work alternately with the teeth of the *escapement wheel* k. This wheel being acted on by the weight tends to move continuously, let us say, in the direction indicated by the arrow-head. Now if the pendulum is at rest, the wheel is held at rest by the pallet m, and with it the whole of the clockwork and the weight. If, however, the pendulum moves and takes the position shown by the dotted line, m is raised, the wheel *escapes* from the confinement in which it was held by the pallet, the weight descends, and causes the wheel to turn until its motion is arrested by the other pallet n; which in consequence of the motion of the pendulum will be brought into contact

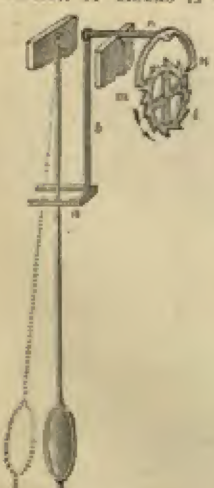

Fig. 59.

with another tooth of the escapement wheel. In this manner the descent of the weight is alternately permitted and arrested—or, in a word, *regulated*—by the pendulum. By means of a proper train of wheelwork the motion of the escapement is communicated to the hands of the clock; and consequently their motion, also, is regulated by the pendulum.

The pendulum has also been used for measuring great velocities. A large block of wood weighing from 3 to 5 tons is coated with iron; against this arrangement, which is known as a *ballistic-pendulum*, a shot is fired, and the deflection thereby produced is observed. From the laws of the impact of inelastic bodies, and from those of the pendulum, the velocity of the ball may be calculated from the amount of this deflection.

The gun may also be fastened to a pendulum arrangement; and, when fired, the reaction causes an angular velocity, from which the pressure of the enclosed gases can be deduced, and therefrom the initial velocity of the shot.

83. **Causes which modify the intensity of terrestrial gravitation.**—The intensity of the force of gravity—that is, the value of g—is not the same in all parts of the earth. It is modified by several causes, of which the form of the earth and its rotation are the most important.

i. The attraction which the earth exerts upon a body at its surface is the sum of the partial attractions which each part of the earth exerts upon that body, and the resultant of all these attractions may be considered to act from a single point, the centre. Hence, if the earth were a perfect sphere, a given body would be equally attracted at any part of the earth's surface. The attraction would, however, vary with the height above the surface. For small alterations of level the differences would be inappreciable; but for greater heights and in accurate measurements observations of the value of g must be reduced to the sea level. The attraction of gravitation being inversely as the square of the distance from the centre (67) we shall have $g : g_t = \frac{1}{R^2} : \frac{1}{(R+h)^2}$ where g is the value of the acceleration of gravity at the sea level, g_t its value at any height h, and R is the radius of the earth. From this, seeing that h is very small compared with R, and that therefore its square may be neglected, we get by simple algebraical transformation

$$g_t = \frac{g}{1 + \frac{2h}{R}} \text{ or } g_t = \frac{gR}{R + 2h}.$$

But even at the sea level the force of gravity varies in different parts in consequence of the form of the earth. The earth is not a true sphere but an ellipsoid, the major axis of which is 12,754,796 metres, and the minor 12,712,160 metres. The distance, therefore, at the centre being greater at the equator than at the Poles, and as the attraction on a body is inversely as the square of these distances, calculation shows that the attraction due to this cause is $\frac{1}{212}$th greater at the Poles than at the equator. This is what would be true if, other things being the same, the earth were at rest.

ii. In consequence of the earth's rotation, the force of gravity is further modified. If we imagine a body relatively at rest on the equator, it really shares the earth's rotation, and describes, in the course of one day, a circle whose centre and radius are the centre and radius of the earth. Now since

a body in motion tends by reason of its inertia to move in a straight line, it follows that to make it move in a circle, a force must be employed at each instant to deflect it from the tangent (53). Consequently, a certain portion of the earth's attraction must be employed in keeping the above body on the surface of the earth, and only the remainder is sensible as *weight* or *accelerating force*. It appears from calculation that on the equator the $\frac{1}{289}$th part of the earth's attraction on any body is thus employed, so that the magnitude of g at the equator is less by the $\frac{1}{289}$th part of what it would be were the earth at rest.

iii. As the body goes nearer the Poles the force of gravity is less and less diminished by the effect of centrifugal force. For in any given latitude it will describe a circle coinciding with the parallel of latitude in which it is placed; but as the radii of these circles diminish, so does the centrifugal force until the Pole, where the radius is null. Further, on the equator the centrifugal force is directly opposed to gravitation; in any other latitude only a component of the whole force is thus employed. This is seen in figure 60, in which PP′ represents the axis of rotation of the earth and EE′ the equator. At any given point E on the equator the centrifugal force is directed along CE, and acts wholly in diminishing the intensity of gravitation; but on any other point, a, nearer the Pole, the centrifugal force acting on a right line ab at right angles to the axis PP′, while gravity acts along aC, gravity is no longer directly diminished by centrifugal force, but only by its component ad, which is less the nearer a is to the Pole.

Fig. 60.

The combined effect of these two causes—the flattening of the earth at the Poles, and the centrifugal force—is to make the attraction of gravitation at the equator less by about the $\frac{1}{192}$ part of its value at the Poles.

CHAPTER III.

MOLECULAR FORCES.

84. **Nature of molecular forces.**—The various phenomena which bodies present show that their molecules are under the influence of two contrary forces, one of which tends to bring them together, and the other to separate them from each other. The first force, which is called *molecular attraction*, varies in one and the same body with the distance only. The second force is due to the vis viva or moving force, which the molecules possess. It is the mutual relation between these forces, the preponderance of the one or the other, which determines the molecular state of a body (4)—whether it be solid, liquid, or gaseous.

Molecular attraction is only exerted at infinitely small distances. Its effect is inappreciable when the distance between the molecules is appreciable.

According to the manner in which it is regarded, molecular attraction is designated by the terms, *cohesion*, *affinity*, or *adhesion*.

85. **Cohesion.**—*Cohesion* is the force which unites adjacent molecules of the same nature; for example, two molecules of water, or two molecules of iron. Cohesion is strongly exerted in solids, less strongly in liquids, and scarcely at all in gases. Its strength decreases as the temperature increases, because then the vis viva of the molecules increases. Hence it is that when solid bodies are heated they first liquefy, and are ultimately converted into the gaseous state, provided that heat produces in them no chemical change.

Cohesion varies not only with the nature of bodies, but also with the arrangement of their molecules; for example, the difference between tempered and untempered steel is due to a difference in the molecular arrangement produced by tempering. Many of the properties of bodies, such as tenacity, hardness, and ductility, are due to the modifications which this force undergoes.

In large masses of liquids, the force of gravity overcomes that of cohesion. Hence liquids acted upon by the former force have no special shape; they take that of the vessel in which they are contained. But in smaller masses cohesion gets the upper hand, and liquids present then the spheroidal form. This is seen in the drops of dew on the leaves of plants; it is also seen when a liquid is placed on a solid which it does not moisten; as, for example, mercury upon wood. The experiment may also be made with water, by sprinkling upon the surface of the wood some light powder such as lycopodium or lampblack, and then dropping some water on it. The following pretty experiment is an illustration of the force of cohesion causing a liquid to assume the spheroidal form. A saturated solution of sulphate of zinc is placed in a

narrow-necked bottle, and a few drops of bisulphide of carbon, coloured with iodine, made to float on the surface. If pure water be now carefully added, so as to rest on the surface of the sulphate of zinc solution the bisulphide collects in the form of a flattened spheroid, which presents the appearance of blown coloured glass, and is larger than the neck of the bottle, provided a sufficient quantity has been taken.

The force of cohesion of liquids may be measured as follows. A plane, perfectly smooth disc D is suspended horizontally to one scale pan *p* of a delicate balance, and is accurately equipoised. A somewhat wide vessel of liquid is placed below, and the position of the disc regulated by means of the sliding screw S until it just touches the liquid. Weights are then carefully added to the other scale pan until the disc is detached from the liquid. In this way it has been found that the weights required to detach the disc vary with the nature of the liquid; with a disc of 118 mm. in diameter the numbers for water, alcohol, and turpentine were 59·4, 31, and 34 grammes respectively.

Fig. 61.

The results were the same whether the disc was of glass, of copper, or of other metals, and they thus only depend on the nature of the liquid. It is a measure of the cohesion of the liquid, for a layer remains adhering to the disc; hence the weight on the other side does not separate the disc from the liquid, but separates the particles of liquid from each other.

86. **Affinity.**—*Chemical affinity*, or *chemical attraction*, is the force which is exerted between molecules not of the same kind. Thus, in water, which is composed of oxygen and hydrogen, it is affinity which unites these elements, but it is cohesion which binds together two molecules of water. In compound bodies cohesion and affinity operate simultaneously, while in simple bodies or elements cohesion has alone to be considered.

To affinity are due all the phenomena of combustion, and of chemical combination and decomposition.

The causes which tend to weaken cohesion are most favourable to affinity; for instance, the action of affinity between substances is facilitated by their division, and still more by reducing them to a liquid or gaseous state. It is most powerfully exerted by a body in its *nascent* state—that is, the state in which the body exists at the moment it is disengaged from a compound; the body is then free, and ready to obey the feeblest affinity. An increase of temperature modifies affinity differently under different circumstances. In some cases, by diminishing cohesion, and increasing the distance between the molecules, heat promotes combination. Sulphur and oxygen, which at the ordinary temperature are without action on each other, combine to form sulphurous acid when the temperature is raised: in other cases heat tends to decompose compounds by imparting to their elements an unequal expansibility. Thus it is that many metallic oxides, as for example those of

silver and mercury, are decomposed, by the action of heat, into gas and metal.

87. **Adhesion.**—The molecular attraction exerted between the *surfaces* of bodies in contact is called *adhesion.*

i. Adhesion takes place between solids. If two leaden bullets are cut with a penknife so as to form two equal and brightly polished surfaces, and the two faces are pressed and turned against each other, until they are in the closest contact, they adhere so strongly as to require a force of more than 100 grammes to separate them. The same experiment may be made with two equal pieces of glass which are polished and made perfectly plane. When they are pressed one against the other, the adhesion is so powerful that they cannot be separated without breaking. As the experiment succeeds *in vacuo*, it cannot be due to atmospheric pressure, but must be attributed to a reciprocal action between the two surfaces. The attraction also increases as the contact is prolonged, and is greater in proportion as the contact is closer.

In the operation of gluing the adhesion is complete, for the pores and crevices of the fresh surfaces being filled with liquid glue, so that there is no empty space on drying, wood and glue form one compact whole. In some cases the adhesion of the cement is so powerful that the mass breaks more readily at other places than at the cemented parts.

There is no real difference between adhesion and cohesion; thus, when two freshly cut surfaces of caoutchouc are pressed together, they adhere with considerable force, and ultimately form one compact solid mass.

ii. Adhesion also takes place between solids and liquids. If we dip a glass rod into water, on withdrawing it a drop will be found to collect at its lower extremity, and remain suspended there. As the weight of the drop tends to detach it, there must necessarily be some force superior to this weight which maintains it there; this force is the force of adhesion.

The adhesion between liquids and solids is more powerful than that between solids. Thus, if in the above experiment a thin layer of oil is interposed between the plates they adhere firmly, but when pulled asunder each plate is moistened by the oil, thus showing that in separating the plates the cohesion of the plates is overcome, but not the adhesion of the oil to the metal. Alcohol adheres more firmly to glass than water. A layer of water on a glass plate is displaced by a drop of alcohol brought on it.

iii. The force of adhesion operates, lastly, between solids and gases. If a glass or metal plate be immersed in water, bubbles will be found to appear on the surface. As air cannot penetrate into the pores of the plate, the bubbles could not arise from the air which had been expelled. It is solely due to the layer of air which covered the plate, and *moistened* it like a liquid. In many cases when gases are separated in the *nascent state* on the surface of metals—as in electrolysis—the layer of gas which covers the plate has such a density that it can produce chemical actions more powerful than those which it can bring about in the free state.

The collection of dust on walls, writing and drawing with chalks and pencils, depend on the adhesion of solids. Yet these are easily rubbed out, for the adhesion is only to the surface layer. In writing with ink, and in water-colour painting, the liquid penetrates into the pores, taking the solid with it which is left behind as the liquid evaporates, and hence the adhesion of such writing and painting is more complete.

CHAPTER IV.

PROPERTIES PECULIAR TO SOLIDS.

88. **Various special properties.**—After having described the principal properties common to solids, liquids, and gases, we shall discuss the properties peculiar to solids. They are, *elasticity of traction*, *elasticity of torsion*, *elasticity of flexure*, *tenacity*, *ductility*, and *hardness*.

89. **Elasticity of traction.**—Elasticity, as a general property of matter, has been already mentioned (17), but simply in reference to the elasticity developed by pressure; in solids it may also be called into play by traction, by torsion, and by flexure. The definitions there given require some extension. In ordinary life we consider those bodies as highly elastic, which, like caoutchouc, undergo considerable change on the application of only a small force. Yet the force of elasticity is greatest in many bodies, such as iron, which do not seem to be very elastic. For by *force of elasticity* is understood the force with which the displaced particles tend to revert to their original position, and which force is equivalent to that which has brought about the change. Considered from this point of view, gases have the least force of elasticity; that of liquids is considerably greater, and is, indeed, greater than that of many solids. Thus, the force of elasticity of mercury is greater than that of caoutchouc, glass, wood, and stone. It is, however, less than that of the other metals, with the exception of lead.

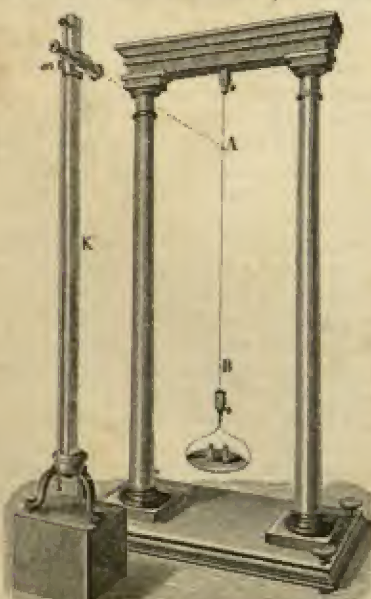

Fig. 62.

This seems discordant with ordinary ideas about elasticity; but it must be remembered that those bodies which, by the exertion of a small force, undergo a considerable change, generally have also the property of undergoing this change without losing the property of reverting completely to their original state. They

have a wide *limit of elasticity* (17). Those bodies which require great force to effect a change are also, for the most part, those on which the exertion of a force produces a permanent alteration; when the force is no longer exerted, they do not completely revert to their original state.

In order to study the laws of the elasticity of traction, Savart used the apparatus represented in fig. 62. It consists of a wooden support from which are suspended the rods or wires taken for experiment. At the lower extremity there is a scale pan, and on the wire two points, A and B, are marked, the distance between which is measured by means of the *cathetometer* before the weights are added.

The *cathetometer* consists of a strong brass support, K, divided into millimetres, and which can be adjusted in a vertical position by means of levelling screws and the plumb line. A small telescope, exactly at right angles to the scale, can be moved up and down, and is provided with a vernier which measures fiftieths of a millimetre. By fixing the telescope successively on the two points A and B, as represented in the figure, the distance between these points is obtained on the graduated scale. Placing then weights in the pan, and measuring again the distance from A to B, the elongation is obtained.

By experiments of this kind it has been ascertained that for elasticity of traction or pressure—

The alteration in length, within the limits of elasticity, is in proportion to the length and to the load acting on the body, and is inversely as the section.

It depends, moreover, on the *specific elasticity*; that is, on the material of the body. If this coefficient be denoted by E, and if the length, section, and load are respectively designated by l, s, and P, then for the alteration in length, e, we have

$$e = E\frac{lP}{s}.$$

If in the above expression the sectional area be a square millimetre, and P be one kilogramme, then

$$e = El, \text{ from which } E = \frac{e}{l},$$

which expresses by what fraction the length of a bar a square millimetre in section is altered by a load of a kilogramme. This is called the *coefficient of elasticity*; it is a very small fraction, and it is therefore desirable to use its reciprocal, that is $\frac{1}{e}$ or μ, as the *modulus of elasticity*; or the weight in kilogrammes which applied to a bar would elongate it by its own length, assuming it to be perfectly elastic. This cannot be observed, for no body is perfectly elastic, but it may be calculated from any accurate observations by means of the above formula.

The following are the best values for some of the principal substances:—

Steel	21,000	Lead	1,800
Wrought Iron	19,000	Wood	1,100
Copper	12,400	Whalebone	700
Brass	9,000	Ice	236
Zinc	8,700	Glass	90
Silver	7,400		

Thus, to double the length of a wrought-iron wire a square millimetre in section, would (if this were possible) require a weight of 19,000 kilogrammes; but a weight of 15 kilogrammes produces a permanent alteration in length of $\frac{1}{1334}$th, and this is the limit of elasticity. The weight which when applied to a body of the unit of section just brings about an appreciable permanent change is a measure of the limit of elasticity. Whalebone, on the contrary, has only a modulus of 700, and experiences a permanent change by a weight of 5 kilogrammes; its limit is, therefore, relatively greater than that of iron. Steel has a high modulus, along with a wide limit.

Both calculation and experiment show that when bodies are lengthened by traction their volume increases.

When weights are placed on a bar, the amount by which it is shortened, or the *coefficient of contraction*, is equal to the elongation which it would experience if the same weights were suspended to it, and is represented by the above numbers.

The influence of temperature on the elasticity of iron, copper, and brass was investigated by Kohlrausch and Loomis. They found that the alteration in the coefficient of elasticity by heat is the same as that which heat produces in the coefficient of expansions and in the refractive power; it is also much the same as the change in the permanent magnetism, and in the specific heat, while it is less than the alteration in the conductivity for electricity.

90. **Elasticity of Torsion.**—The laws of the torsion of wires were determined by Coulomb, by means of an apparatus called the *torsion balance* (fig. 63). It consists of a metal wire, clasped at its upper extremity in a support, A, and holding at the other extremity a metal sphere, B, to which is affixed an index, C. Immediately below this there is a graduated circle, CD. If the needle is turned from its position of equilibrium through a certain angle, which is the *angle of torsion*, the force necessary to produce this effect is the *force of torsion*. When, after this deflection, the sphere is left to itself, the reaction of torsion produces its effect, the wire untwists itself, and the sphere rotates about its vertical axis with increasing rapidity until it reaches its position of equilibrium. It does not, however, rest there; in virtue of its inertia it passes this position, and the wire undergoes a torsion in the opposite direction. The equilibrium being again destroyed, the wire again tends to untwist itself, the same alterations are again produced, and the needle does not rest at zero of the scale until after a certain number of oscillations about this point have been completed.

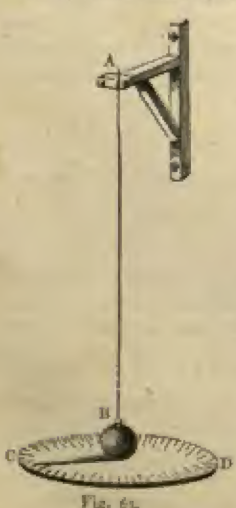

Fig. 63.

By means of this apparatus Coulomb found that when the amplitude of the oscillations is within certain limits, the oscillations are subject to the following laws:

I. *The oscillations are very nearly isochronous.*

II. *For the same wire, the angle of torsion is proportional to the moment of the force of torsion.*

III. *With the same force of torsion, and with wires of the same diameter, the angles of torsion are proportional to the lengths of the wires.*

IV. *The same force of torsion being applied to wires of the same length, the angles of torsion are inversely proportional to the fourth powers of the diameters.*

Wertheim has examined the elasticity of torsion in the case of stout rods by means of a different apparatus, and finds that it is also subject to these laws. He has further found that, all dimensions being the same, different substances undergo different degrees of torsion, and each substance has its own coefficient of torsion, which is denoted by $\frac{1}{T}$.

The laws of torsion may be enunciated in the formula $w = \frac{1}{T}\frac{Fl}{r^4}$; in which w is the angle of torsion, F the moment of the force of torsion, l the length of the wire, r its diameter, and $\frac{1}{T}$ the specific torsion-coefficient.

91. **Elasticity of flexure.**—A solid, when cut into a thin plate, and fixed at one of its extremities, after having been more or less bent, strives to return to its original position when left to itself. This property is the elasticity of flexure, and is very distinct in steel, caoutchouc, wood, and paper.

If a rectangular bar A B be clamped at one end and loaded at the other (fig. 64), the flexure e is represented by the formula

$$e = \frac{Wl^3}{bh^3\mu}$$

where W is the load, l the length of the bar, b its breadth, h its thickness, and μ the modulus of elasticity.

The elasticity of flexure is applied in a vast variety of instances—for example, in bows, watch springs, carriage springs; in spring balances it is used to determine weights, in dynamometers to determine the force of agents in prime movers; and, as existing in wool, hair, and feathers, it is applied to domestic uses in cushions and mattresses.

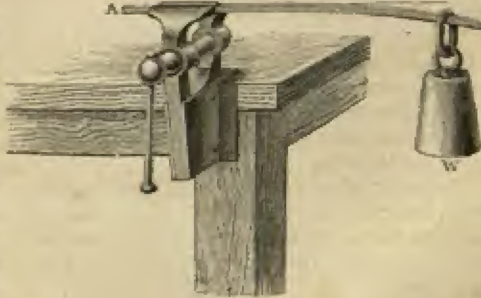

Fig. 64.

Whatever be the kind of elasticity, there is, as has been already said, a limit to it—that is, there is a molecular displacement, beyond which bodies are broken, or at any rate do not regain their primitive form. This limit is affected by various causes. The elasticity of many metals is increased by *hardening*, whether by cold, by means of the draw-plate, by rolling, or by hammering.

Some substances, such as steel, cast iron, and glass, become both harder and more elastic by tempering (95).

Elasticity, on the other hand, is diminished by *annealing*, which consists in raising the body to a temperature lower than that necessary for tempering, and allowing it to cool slowly. It is by this means that the elasticity of springs may be regulated at pleasure. Glass, when it is heated, undergoes a true tempering in being rapidly cooled, and hence, in order to lessen the fragility of glass objects, they are reheated in a furnace, and are carefully allowed to cool slowly, so that the particles have time to assume their most stable position (95).

92. **Tenacity.**—*Tenacity* is the resistance which a body opposes to the total separation of its parts. According to the manner in which the external force acts, we may have various kinds of tenacity: *tenacity* in the ordinary sense, or resistance to traction; *relative* tenacity, or resistance to fracture; *reactive* tenacity, or resistance to crushing; *sheering* tenacity, or resistance to displacement of particles in a lateral direction; and *torsional* tenacity, or resistance to twisting. Ordinary tenacity is determined in different bodies by forming them into cylindrical or prismatic wires, and ascertaining the weight necessary to break them.

Mere increase in length does not influence the breaking weight, for the weight acts in the direction of the length, and stretches all parts as if it had been directly applied to them.

Tenacity is directly proportional to the breaking weight, and inversely proportional to the area of a transverse section of the wire.

Tenacity diminishes with the duration of the traction. A small force continuously applied for a long time will often break a wire, which would not at once be broken by a larger weight.

Not only does tenacity vary with different substances, but it also varies with the form of the body. Thus, with the same sectional area, a cylinder has greater tenacity than a prism. The quantity of matter being the same, a hollow cylinder has greater tenacity than a solid one; and the tenacity of this hollow cylinder is greatest when the external radius is to the internal one in the ratio of 11 to 5.

The shape has also the same influence on the resistance to crushing as it has on the resistance to traction. A hollow cylinder with the same mass, and the same weight, offers a greater resistance than a solid cylinder. Thus it is that the bones of animals, the feathers of birds, the stems of corn and other plants, offer greater resistance than if they were solid, the mass remaining the same.

Tenacity, like elasticity, is different in different directions in bodies. In wood, for example, both the tenacity and the elasticity are greater in the direction of the fibres than in a transverse direction. And this difference obtains in general in all bodies, the texture of which is not the same in all directions.

Wires by being worked acquire greater tenacity on the surface, and have therefore a higher coefficient, than even somewhat thicker rods of the same material. A strand of wires is stronger than a rod of the same section.

Wertheim found the following numbers representing the weight in kilo-

grammes for the limit of elasticity and for the tenacity of wires, 1 mm. in diameter.

The table shows that of all metals cast steel has the greatest tenacity. Yet it is exceeded by fibres of unspun silk, a thread of which 1 square millimetre in section can carry a load of 500 kilogrammes. Single fibres of cotton can support a weight of 100 to 300 grammes; that is, millions of times their own weight.

		Limit of Elasticity. Kilogrammes.	Tenacity. Kilogrammes.
Lead	drawn	0·25	2·07
	annealed	0·20	1·80
Tin	drawn	0·45	2·45
	annealed	0·20	1·70
Gold	drawn	13·50	27·00
	annealed	3·00	10·08
Silver	drawn	11·25	29·00
	annealed	2·75	16·02
Zinc	drawn	0·75	12·80
	annealed	1·00	
Copper	drawn	12·00	40·30
	annealed	3·00	30·54
Platinum	drawn	26·00	34·10
	annealed	14·50	23·50
Iron	drawn	32·5	61·10
	annealed	5·0	46·88
Steel	drawn	42·5	70·00
	annealed	15·0	40·00
Cast Steel	drawn	55·6	80·00
	annealed	5·0	65·75

In this table the bodies are supposed to be at the ordinary temperature. At higher temperatures the tenacity rapidly decreases. Seguin made some experiments on this point with iron and copper, and obtained the following values for the tenacity, in kilogrammes, of millimetre wire at different temperatures :—

Iron . . at 10°, 60; at 370°, 54; at 500°, 37;
Copper . . " 21; " 7·7; " 0.

93. **Ductility.**—*Ductility* is the property in virtue of which a great number of bodies change their forms by the action of traction or pressure.

With certain bodies, such as clay, wax, &c., the application of a very little force is sufficient to produce a change; with others, such as the resins and glass, the aid of heat is needed, while with the metals more powerful agents must be used, such as percussion, the draw-plate, or the rolling-mill.

Malleability is that modification of ductility which is exhibited by hammering. The most malleable metal is gold, which has been beaten into leaves about the $\frac{1}{360000}$th of an inch thick.

The most ductile metal is platinum. Wollaston obtained a wire of it 0·00003 of an inch in diameter. This he effected by covering with silver a platinum wire 0·01 of an inch in diameter, so as to obtain a cylinder 0·2 inch

in diameter only, the axis of which was of platinum. This was then drawn out in the form of wire as fine as possible; the two metals were equally extended. When this wire was afterwards boiled with dilute nitric acid the silver was dissolved, and the platinum wire left intact. The wire was so fine that a mile of it would have only weighed 1·25 of a grain.

94. **Hardness.**—*Hardness* is the resistance which bodies offer to being scratched or worn by others. It is only a relative property, for a body which is hard in reference to one body may be soft in reference to others. The relative hardness of two bodies is ascertained by trying which of them will scratch the other. Diamond is the hardest of all bodies, for it scratches all, and is not scratched by any. The hardness of a body is expressed by referring it to a *scale of hardness*: that usually adopted is—

1. Talc	5. Apatite	8. Topaz
2. Rock salt	6. Felspar	9. Corundum
3. Calcspar	7. Quartz	10. Diamond
4. Fluorspar		

Thus, the hardness of a body which would scratch felspar, but would be scratched by quartz, would be expressed by the number 6·5.

The pure metals are softer than their alloys. Hence it is that, for jewellery and coinage, gold and silver are alloyed with copper to increase their hardness.

The hardness of a body has no relation to its resistance to compression. Glass and diamond are much harder than wood, but the latter offers far greater resistance to the blow of a hammer. Hard bodies are often used for polishing powders; for example, emery, pumice, and tripoli. Diamond being the hardest of all bodies, can only be ground by means of its own powder.

A body which moves with great velocity can cut into bodies which are harder than itself. Thus a disc of wrought iron rotating with a velocity of 11 metres in a second was cut by a steel graver; while when it rotated with a velocity of 20 metres, the edge of the disc could cut the graver, and with a velocity of 50 to 100 metres, it could even cut into agate and quartz.

95. **Temper.**—By sudden cooling after they have been raised to a high temperature, many bodies acquire great hardness. This operation is called *tempering*. All cutting instruments are made of tempered steel. There are, however, some few bodies upon which tempering produces quite a contrary effect. An alloy of one part of tin and four parts of copper, called *tamtam metal*, is ductile and malleable when rapidly cooled, but hard and brittle as glass when cooled slowly.

BOOK III.

ON LIQUIDS.

CHAPTER I.

HYDROSTATICS.

96. **Object of Hydrostatics.**—The science of *hydrostatics* treats of the conditions of the equilibrium of liquids, and of the pressures they exert, whether within their own mass or on the sides of the vessels in which they are contained.

The science which treats of the motion of liquids is *hydrodynamics*, and the application of the principles of this science to conducting and raising water in pipes is known by the name of *hydraulics*.

97. **General characters of liquids.**—It has been already seen (4) that liquids are bodies whose molecules are displaced by the slightest force. Their fluidity, however, is not perfect; their particles always adhere slightly to each other, and when a thread of liquid moves, it attempts to drag the adjacent stationary particles with it, and conversely is held back by them. This property is called *viscosity*.

Gases also possess fluidity, but in a higher degree than liquids. The distinction between the two forms of matter is that liquids are almost incompressible and are comparatively inexpansible, while gases are eminently compressible and expand spontaneously.

The fluidity of liquids is seen in the readiness with which they take all sorts of shapes. Their compressibility is established by the following experiment.

98. **Compressibility of liquids.**—From the experiment of the Florentine Academicians (13), liquids were for a long time regarded as being completely incompressible. Since then, researches have been made on this subject by various physicists, which have shown that liquids are really compressible.

The apparatus used for measuring the compressibility of liquids has been named the *piezometer* (πιέζω, I compress, μέτρον, measure). That shown in fig. 65 consists of a strong glass cylinder, with very thick sides, and an internal diameter of about 3¼ inches. The base of the cylinder is firmly cemented into a wooden foot, and on its upper part is fitted a metallic cylinder closed by a cap which can be unscrewed. In this cap there is a funnel,

R, for introducing water into the cylinder, and a small barrel hermetically closed by a piston, which is moved by a screw, P.

In the inside of the apparatus there is a glass vessel, A, containing the liquid to be compressed. The upper part of this vessel terminates in a capillary tube, which dips under mercury, O. This tube has been previously divided into parts of equal capacity, and it has been determined how many of these parts the vessel A contains. The latter is ascertained by finding the weight, P, of the mercury which the reservoir, A, contains, and the weight, p, of the mercury contained in a certain number of divisions, n, of the capillary tube. If N be the number of divisions of the small tube contained in the whole reservoir, we have $\frac{N}{n}=\frac{P}{p}$, from which the value of N is obtained. There is further a *manometer*. This is a glass tube, B, containing air, closed at one end, and the other end of which dips under mercury. When there is no pressure on the water in the cylinder, the tube B is completely full of air; but when the water within the cylinder is compressed by means of the screw P, the pressure is transmitted to the mercury, which rises in the tube, compressing the air which it contains. A graduated scale fixed on the side of the tube shows the reduction of volume, and this reduction of volume indicates the pressure exerted on the liquid in the cylinder, as will be seen in speaking of the manometer (177).

Fig. 65.

In making the experiment, the vessel A is filled with the liquid to be compressed, and the end dipped under the mercury. By means of the funnel R the cylinder is entirely filled with water. The screw P being then turned the piston moves downwards, and the pressure exerted upon the water is transmitted to the mercury and the air; in consequence of which the mercury rises in the tube B, and also in the capillary tube. The ascent of mercury in the capillary tube shows that the liquid in the vessel A has diminished in volume, and gives the amount of its compression, for the capacity of the whole vessel A in terms of the graduated divisions on the capillary tube has been previously determined.

In his first experiments, Oersted assumed that the capacity of the vessel A remained the same, its sides being compressed both internally and externally by the liquid. But mathematical analysis proves that this capacity diminishes in consequence of the external and internal pressures. Colladon and Sturm have made some experiments allowing for this change of capacity, and have found that for a pressure equal to that of the atmosphere, mercury experiences a compression of 0·000005 parts of its original volume, water a compression of 0·00005, and ether a compression of 0·000133 parts of its

original bulk. The compressibility of sea water is only about 0·000044: it is not materially denser even at great depths; thus at the depth of a mile its density would only be about $\frac{1}{130}$th the greater. The compressibility is greater the higher the original temperature; thus that of ether at 14° is one-fourth greater than its compressibility at 0°.

For *water and mercury* it was also found that within certain limits the decrease of volume is proportional to the pressure.

Whatever be the pressure to which a liquid has been subjected, experiment shows that as soon as the pressure is removed the liquid regains its original volume, from which it is concluded that *liquids are perfectly elastic.*

99. **Equality of pressures. Pascal's law.**—By considering liquids as perfectly fluid, and assuming them to be uninfluenced by the action of gravity, the following law has been established. It is often called Pascal's law, for it was first enunciated by him.

Pressure exerted anywhere upon a mass of liquid is transmitted undiminished in all directions, and acts with the same force on all equal surfaces, and in a direction at right angles to those surfaces.

To get a clearer idea of the truth of this principle, let us conceive a vessel of any given form in the sides of which are placed various cylindrical apertures, all of the same size, and closed by movable pistons. Let us, further, imagine this vessel to be filled with liquid and unaffected by the action of gravity; the pistons will, obviously, have no tendency to move. If now upon the piston A (fig. 66), which has a surface *a*, a weight of P pounds be placed, it will be pressed inwards, and the pressure will be transmitted to the internal faces of each of the pistons, B, C, D, and E, which will each be forced outwards by a pressure P, their surfaces being equal to that of the first piston. Since each of the pistons undergoes a pressure P, equal to that on A, let us suppose two of the pistons united so as to constitute a surface 2*a*, it will have to support a pressure 2P. Similarly, if the piston were equal to 3*a*, it would experience a pressure of 3P; and if its area were 100 or 1,000 times that of *a*, it would sustain a pressure of 100 or 1,000 times P. In other words, the pressure on any part of the internal walls of the vessel would be proportional to the surface.

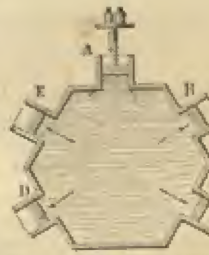

Fig. 66.

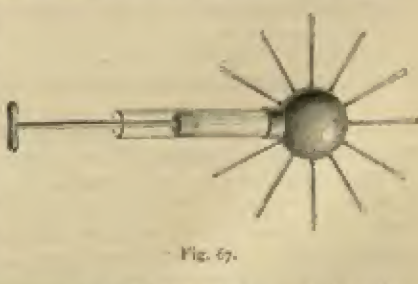
Fig. 67.

The principle of the equality of pressure is assumed as a consequence of the constitution of fluids. By the following experiment it can be shown that pressure is transmitted in all directions, although it cannot be shown that it is equally transmitted. A cylinder provided with a piston is fitted into a hollow sphere (fig. 67), in which

small cylindrical jets are placed perpendicular to the sides. The sphere and the cylinder being both filled with water, when the piston is moved the liquid spouts forth from all the orifices, and not merely from that which is opposite to the piston.

The reason why a satisfactory quantitative experimental demonstration of the principle of the equality of pressure cannot be given is, that the influence of the weight of the liquid and of the friction of the pistons cannot be eliminated.

Yet an approximate verification may be effected by the experiment represented in fig. 68. Two cylinders of different diameters are joined by a tube and filled with water. On the surface of the liquid are two pistons P and *p*, which hermetically close the cylinders, but move without friction. Let the area of the large piston be, for instance, thirty times that of the smaller one. That being assumed, let a weight, say of two pounds, be placed upon the small piston; this pressure will be transmitted to the water and to the large piston, and as this pressure amounts to two pounds *on each portion of its surface equal to that of the small piston*, the large piston must be exposed to an upward pressure thirty times as much, or of sixty pounds. If now this weight be placed upon the large piston, both will remain in equilibrium; but if the weight is greater or less, this is no longer the case. If S and *s* are the areas of the large and small piston respectively, and P and *p* the corresponding loads, then, $\frac{P}{p} = \frac{S}{s}$; whence $P = \frac{pS}{s}$.

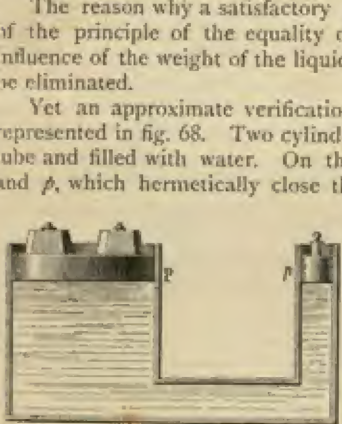

Fig. 68.

It is important to observe that in speaking of the transmission of pressures to the sides of the containing vessel, these pressures must always be supposed to be perpendicular to the sides; for any oblique pressure may be decomposed into two others, one at right angles to the side, and the other acting parallel with the side; but as the latter has no action on the side, the perpendicular pressure is the only one to be considered.

PRESSURE PRODUCED IN LIQUIDS BY GRAVITY.

100. **Vertical downward pressure; its laws.**—Any given liquid being in a state of rest in a vessel, if we suppose it to be divided into horizontal layers of the same density, it is evident that each layer supports the weight of those above it. Gravity, therefore, produces internal pressures in the mass of a liquid which vary at different points. These pressures are submitted to the following general laws:—

I. *The pressure in each layer is proportional to the depth.*

II. *With different liquids and the same depth, the pressure is proportional to the density of the liquid.*

III. *The pressure is the same at all points of the same horizontal layer.*

The first two laws are self-evident; the third necessarily follows from the first and from Pascal's principle.

Meyer has found, by direct experiments, that pressures are transmitted through liquids contained in tubes, with the same velocity as that with which sound travels under the same circumstances.

101. **Vertical upward pressure.**—The pressure which the upper layers of a liquid exert on the lower layers causes them to exert an equal reaction in an upward direction, a necessary consequence of the principle of transmission of pressure in all directions. This upward pressure is termed the *buoyancy* of liquids; it is very sensible when the hand is plunged into a liquid, more especially one of great density, like mercury.

The following experiment (fig. 69) serves to exhibit the upward pressure of liquids. A large open glass tube A, one end of which is ground, is fitted with a ground-glass disc, O, or still better with a thin card or piece of mica, the weight of which may be neglected. To the disc is fitted a string, C, by which it can be held against the bottom of the tube. The whole is then immersed in water, and now the disc does not fall, although no longer held by the string; it is consequently kept in its position by the upward pressure of the water. If water be now slowly poured into the tube, the disc will only sink when the height of the water inside the tube is equal to the height outside. It follows thence that the upward pressure on the disc is equal to the pressure of a column of water, the base of which is the internal section of the tube A, and the height the distance from the disc to the upper surface of the liquid. Hence the *upward pressure of liquids at any point is governed by the same laws as the downward pressure.*

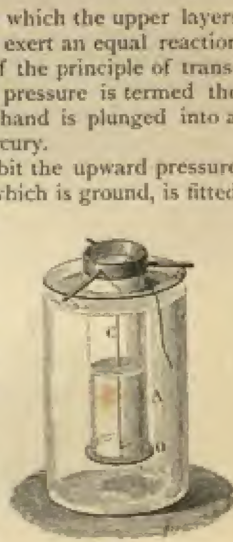

Fig. 69.

102. **Pressure is independent of the shape of the vessel.**—The pressure exerted by a liquid, in virtue of its weight, on any portion of the liquid, or on the sides of the vessel in which it is contained, depends on the depth and density of the liquid, but *is independent of the shape of the vessel and of the quantity of the liquid.*

This principle, which follows from the law of the equality of pressure, may be experimentally demonstrated by many forms of apparatus. The following is the one most frequently used, and is due to Haldat. It consists of a bent tube, ABC (fig. 70), at one end of which, A, is fitted a stop-cock, in which can be screwed two vessels, M and P, of the same height, but different in shape and capacity, the first being conical, and the other nearly cylindrical. Mercury is poured into the tube, ABC, until its level nearly reaches A. The vessel M is then screwed on and filled with water. The pressure of the water acting on the mercury causes it to rise in the tube C, and its height may be marked by means of a little collar, *a*, which slides up and down the tube. The level of the water in M is also marked by means of the movable rod *o*. When this is done, M is emptied by means of the stop-cock, unscrewed, and replaced by P. When water is now poured in this, the mercury, which had resumed its original level in the tube ABC,

again rises in C, and when the water in P has the same height as it had in M, which is indicated by the rod *o*, the mercury will have risen to the height it had before, which is marked by the collar *a*. Hence the pressure on the mercury in both cases is the same. This pressure is therefore independent of the shape of the vessels, and, consequently, also of the quantity

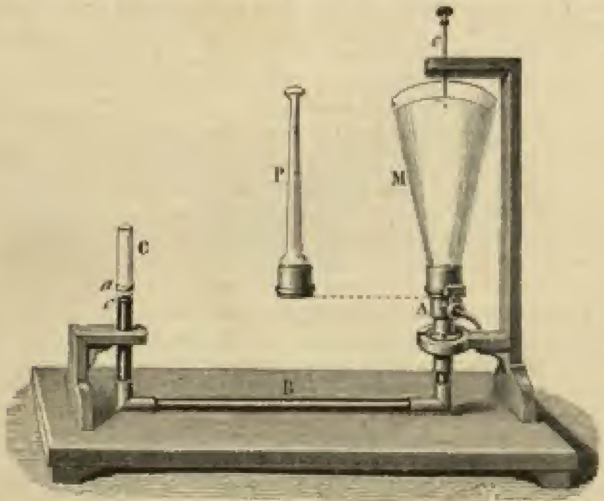

Fig. 70.

of liquid. The base of the vessel is obviously the same in both cases; it is the surface of the mercury in the interior of the tube A.

Another mode of demonstrating this principle is by means of an apparatus devised by Masson. In this (fig. 71) the pressure of the water contained in the vessel M is not exerted upon the column of mercury, as in that of Haldat, but on a small disc or stop *a*, which closes a tubulure *c*, on which is screwed the vessel M. The disc is not fixed to the tubulure, but is sustained by a thread attached to the end of a scale-beam. At the other end is a pan in which weights can be placed until they counterbalance the pressure exerted by the water on the stop. The vessel M being emptied is unscrewed, and replaced by the narrow tube O. This being filled to the same height as the large vessel, which is observed by means of the mark *o*, it will be observed that to keep the disc in its place just the same weight must be placed in the pan as before, which leads, therefore, to the same conclusion as does Haldat's experiment. The same result is obtained if, instead of the vertical tube P, the oblique tube Q be screwed to the tubulure.

From a consideration of these principles it will be readily seen that a very small quantity of water can produce considerable pressures. Let us imagine any vessel—a cask, for example—filled with water and with a long narrow tube tightly fitted into the side. If water is poured into the tube, there will be a pressure on the bottom of the cask equal to the weight of a column of water whose base is the bottom itself, and whose height is equal

to that of the water in the tube. The pressure may be made as great as we please; by means of a narrow thread of water forty feet high, Pascal succeeded in bursting a very solidly constructed cask.

The toy known as the *hydrostatic bellows* depends on the same principle, and we shall shortly see a most important application of it in the hydraulic press.

From the principle just laid down, the pressures produced at the bottom of the sea may be calculated. It will be presently demonstrated that the pressure of the atmosphere is equal to that of a column of sea-water about

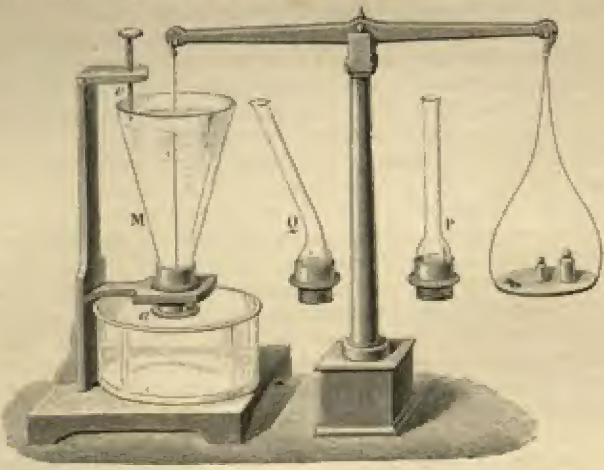

Fig. 71.

thirty-three feet high. At sea the lead has frequently descended to a depth of thirteen thousand feet; at the bottom of some seas, therefore, there must be a pressure of four hundred atmospheres.

103. **Pressure on the sides of vessels.**—Since the pressure caused by gravity in the mass of a liquid is transmitted in every direction, according to the general law of the transmission of fluid pressure, it follows that at every point of the side of any vessel a pressure is exerted, at right angles to the side, which we will suppose to be plane. The resultant of all these pressures is the total pressure on the sides. But since these pressures increase in proportion to the depth, and also in proportion to the horizontal extent of their side, their resultant can only be obtained by calculation, which shows that the total pressure on any given portion of the side *is equal to the weight of a column of liquid, which has this portion of the side for its base, and whose height is the vertical distance from the centre of gravity of the portion to the surface of the liquid.* If the side of a vessel is a curved surface the same rule gives the pressure on the surface, but the total pressure is no longer the resultant of the fluid pressures.

The point in the side supposed plane, at which the resultant of all the pressure is applied, is called the *centre of pressure*, and is always below the

centre of gravity of the side. For if the pressures exerted at different parts of the plane side were equal, the point of application of their resultant, the centre of pressure would obviously coincide with the centre of gravity of the side. But since the pressure increases with the depth, the centre of pressure is necessarily below the centre of gravity. This point is determined by calculation which leads to the following results :—

i. With a rectangular side whose upper edge is level with the water, the centre of pressure is at two-thirds of the line which joins the middle of the horizontal sides measured from the top.

ii. With a triangular side whose base is horizontal, and coincident with the level of the water, the centre of pressure is at the middle of the line which joins the vertex of the triangle with the middle of the base.

iii. With a triangular side whose vertex is level with the water, the centre of pressure is in the line joining the vertex and the middle of the base, and at three-fourths of the distance of the latter from the vertex.

104. **Hydrostatic paradox.**—We have already seen that the pressure on the bottom of a vessel depends neither on the form of the vessel nor on the quantity of the liquid, but simply on the height of the liquid above the bottom. But the pressure thus exerted must not be confounded with the pressure which the vessel itself exerts on the body which supports it. The latter is always equal to the combined weight of the liquid and the vessel in which it is contained, while the former may be either smaller or greater than this weight according to the form of the vessel. This fact is often termed the *hydrostatic paradox*, because at first sight it appears paradoxical.

CD (fig. 72) is a vessel composed of two cylindrical parts of unequal diameters, and filled with water to *a*. From what has been said before, the bottom of the vessel CD supports the same pressure as if its diameter were everywhere the same as that of its lower part ; and it would at first sight seem that the scale MN of the balance, in which the vessel CD is placed, ought to show the same weight as if there had been placed in it a cylindrical vessel having the same height of water, and having the diameter of the part D. But the pressure exerted on the bottom of the vessel is not all transmitted to the scale MN ; for the *upward* pressure upon the surface *no* of the vessel is precisely equal to the weight of the *extra* quantity of water which a cylindrical vessel would contain, and balances an equal portion of the *downward* pressure on *m*. Consequently, the pressure on the plate MN is simply equal to the weight of the vessel CD and of the water which it contains.

Fig. 72.

CONDITIONS OF THE EQUILIBRIUM OF LIQUIDS.

105. **Equilibrium of a liquid in a single vessel.**—In order that a liquid may remain at rest in a vessel of any given form, it must satisfy the two following conditions :—

I. *Its surface must be everywhere perpendicular to the resultant of the forces which act on the molecules of the liquid.*

II. *Every molecule of the mass of the liquid must be subject in every direction to equal and contrary pressures.*

The second condition is self-evident; for if, in two opposite directions, the pressures exerted on any given molecule were not equal and contrary, the molecule would be moved in the direction of the greater pressure, and there would be no equilibrium. Thus the second condition follows from the principle of the equality of pressures, and from the reaction which all pressure causes on the mass of liquids.

To prove the first condition, let us suppose that *mp* is the resultant of all the forces acting upon any molecule *m* on the surface (fig. 73), and that this surface is inclined in reference to the force *mp*. The latter can consequently be decomposed into two forces, *mq* and *mf*; the one perpendicular to the surface of the liquid and the other to the direction *mp*. Now the first force, *mq*, would be destroyed by the resistance of the liquid, while the second would move the molecule in the direction *mf*, which shows that the equilibrium is impossible.

Fig. 73.

If gravity be the force acting on the liquid, the direction *mp* is vertical; hence, if the liquid is contained in a basin or vessel of small extent, the surface ought to be plane and horizontal (68), because then the direction of gravity is the same in every point. But the case is different with liquid surfaces of greater extent, like the ocean. The surface will be perpendicular to the direction of gravity: but as this changes from one point to another, and always tends towards a point near the centre of the earth, it follows that the direction of the surface of the ocean will change also, and assume a nearly spherical form.

106. **Equilibrium of the same liquid in several communicating vessels.**—When several vessels of any given form communicate with each other, there will be equilibrium when the liquid in each vessel satisfies the two preceding conditions (105), and further, *when the surfaces of the liquids in all the vessels are in the same horizontal plane.*

In the vessels ABCD (fig. 74), which communicate with each other, let us consider any transverse section of the tube *mn*; the liquid can only remain in equilibrium as long as the pressures which this section supports from *m* in the direction of *n*, and from *n* in the direction of *m*, are equal and opposite. Now it has been already proved that these pressures are respectively equal to the weight of a column of water, whose base is the supposed section, and whose height is the distance from the centre of gravity of this section to the surface of the liquid. If we conceive, then, a horizontal plane, *mn*, drawn through the centre of gravity of this section, it will be seen that

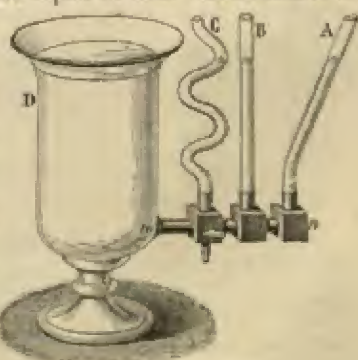

Fig. 74.

there will only be equilibrium as long as the height of the liquid above this plane is the same in each vessel, which demonstrates the principle enunciated.

107. **Equilibrium of superposed liquids.**—In order that there should be equilibrium when several heterogeneous liquids are superposed in the same vessel, each of them must satisfy the conditions necessary for a single liquid (105); and further, *there will be stable equilibrium only when the liquids are arranged in the order of their decreasing densities from the bottom upwards.*

The last condition is experimentally demonstrated by means of the *phial of four elements.* This consists of a long narrow bottle containing mercury, water saturated with carbonate of potass, alcohol coloured red, and petroleum. When the phial is shaken the liquids mix, but when it is allowed to rest they separate; the mercury sinks to the bottom, then comes the water, then the alcohol, and then the petroleum. This is the order of the decreasing densities of the bodies. The water is saturated with carbonate of potass to prevent its mixing with the alcohol.

This separation of the liquids is due to the same cause as that which enables solid bodies to float on the surface of a liquid of greater density than their own. It is also on this account that fresh water, at the mouths of rivers, floats for a long time on the denser salt water of the sea; and it is for the same reason that cream, which is lighter than milk, rises to the surface.

108. **Equilibrium of two different liquids in communicating vessels.**—When two liquids of different densities, which do not mix, are contained in two communicating vessels, they will be in equilibrium when, in addition to the preceding principles, they are subject to the following: *that the heights above the horizontal surface of contact of two columns of liquid in equilibrium are in the inverse ratio of their densities.*

To show this experimentally, mercury is poured into a bent glass tube, *mn*, fixed against an upright wooden support (fig. 75), and then water is poured into one of the legs, AB. The column of water, AB, pressing on the mercury at B, lowers its level in the leg AB, and raises it in the other by a quantity, CD; so that if, when equilibrium is established, we imagine a horizontal plane, BC, to pass through B, the column of water in AB will balance the column of mercury CD. If the heights of these two columns are then measured, by means of the scales, it will be found that the height of the column of water is about $13\frac{1}{2}$ times that of the height of the column of mercury. We shall presently see that the density of mercury is about $13\frac{1}{2}$ times that of water, from which it follows that the heights are inversely as the densities.

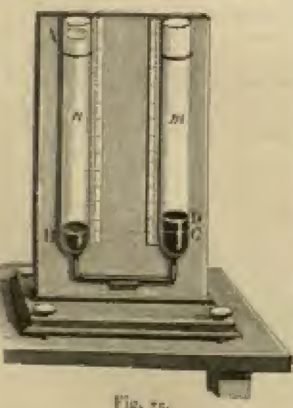

Fig. 75.

It may be added that the equilibrium cannot exist unless there is a sufficient quantity of the heavier liquid for part of it to remain in *both legs* of the tube.

The preceding principle may be deduced by a very simple calculation. Let d and d' be the densities of water and mercury, and h and h' their respective heights, and let g be the force of gravity. The pressure on B will be proportional to the density of the liquid, to its height, and to the force of gravity; on the whole, therefore, to the product dhg. Similarly the pressure at C will be proportional to $d'h'g$. But in order to produce equilibrium, dhg must be equal to $d'h'g$, or $dh = d'h'$. This is nothing more than an algebraical expression of the above principle; for since the two products must always be equal, d' must be as many times greater than d, as h' is less than h.

In this manner the density of a liquid may be determined. Suppose one of the branches contained water and the other oil, and their heights were, respectively, 15 inches for the oil and 14 inches for the water. The density of water being taken as unity, and that of oil being called x, we shall have

$$15 \times x = 14 \times 1 \text{; whence } x = \frac{14}{15} = 0{\cdot}933.$$

APPLICATIONS OF THE PRECEDING HYDROSTATIC PRINCIPLES.

109. **Hydraulic press.**—The law of the equality of pressure has received a most important application in the *hydraulic press*, a machine by which

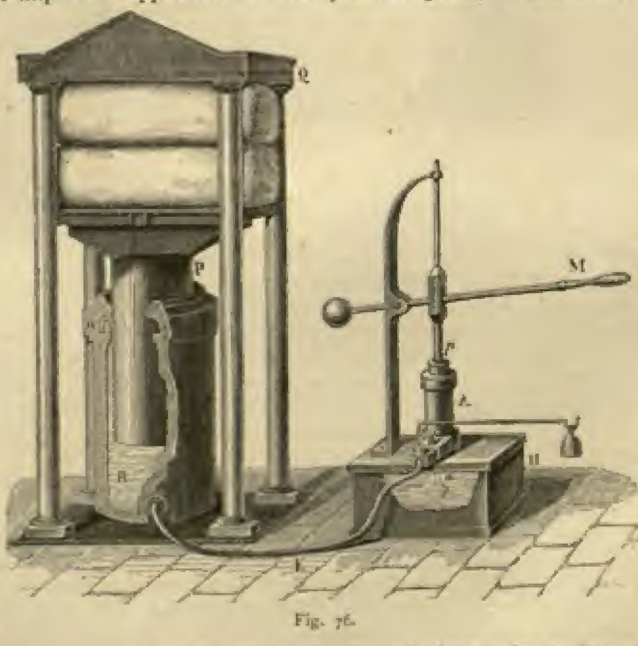

Fig. 76.

enormous pressures may be produced. Its principle is due to Pascal, but it was first constructed by Bramah in 1796.

It consists of a cylinder, B, with very strong thick sides (fig. 76), in

which there is a cast-iron ram, P, working water-tight in the collar of the cylinder. On the ram P there is a cast-iron plate on which the substance to be pressed is placed. Four strong columns serve to support and fix a second plate Q.

By means of a leaden pipe K, the cylinder, B, which is filled with water, communicates with a small force-pump, A, which works by means of a lever, M. When the piston of this pump *p* ascends, a vacuum is produced and the water rises in the tube *a*, at the end of which there is a rose, to prevent the entrance of foreign matters. When the piston *p* descends, it drives the water into the cylinder by the tube K.

Fig. 77 represents a section, on a larger scale, of the system of valves necessary in working the apparatus. The valve *o*, below the piston *p*, opens

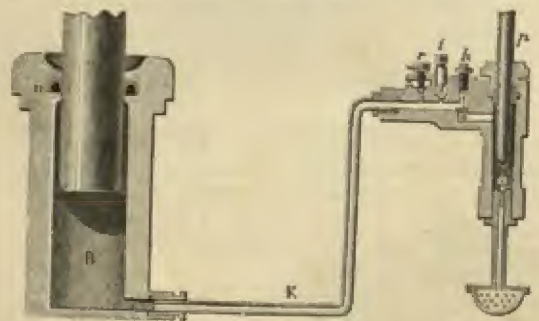

Fig. 77.

when the piston rises, and closes when it descends. The valve *o*, during this descent, is opened by the pressure of the water which passes by the pipe K. The valve *i* is a *safety valve*, held by a weight which acts on it by means of a lever. By weighting the latter to a greater or less extent the pressure can be regulated, for as soon as there is an upward pressure greater than that of the weight upon it, it opens and water escapes. A screw *r* serves to relieve the pressure, for when it is opened it affords a passage for the efflux of the water in the cylinder B.

A most important part is the leather collar, *n*, the invention of which by Bramah removed the difficulties which had been experienced in making the large ram work water-tight when submitted to great pressures. It consists of a circular piece of stout leather, fig. 78, saturated with oil so as to be impervious to water, in the centre of which a circular hole is cut. This piece is bent so that a section of it represents a reversed U, and is fitted into a groove *n* made in the neck of the cylinder. This collar being concave downwards, in proportion as the pressure increases, it fits the more tightly against the ram P on one side and the neck of the cylinder on the other, and quite prevents any escape of water.

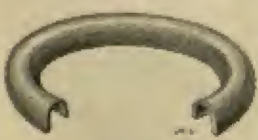

Fig. 78.

The pressure which can be obtained by this press depends on the relation

of the piston P to that of the piston *p*. If the former has a transverse section fifty or a hundred times as large as the latter, the upward pressure on the large piston will be fifty or a hundred times that exerted upon the small one. By means of the lever M an additional advantage is obtained. If the distance from the fulcrum to the point where the power is applied is five times the distance from the fulcrum to the piston *p*, the pressure on *p* will be five times the power. Thus, if a man acts on M with a force of sixty pounds, the force transmitted by the piston *p* will be 300 pounds, and the force which tends to raise the piston P will be 30,000 pounds, supposing the section of P is a hundred times that of *p*.

The hydraulic press is used in all cases in which great pressures are required. It is used in pressing cloth and paper, in extracting the juice of beet-root, in compressing hay and cotton, in expressing oil from seeds, and in bending iron plates; it also serves to test the strength of cannon, of steam boilers, and of chain cables. The parts composing the tubular bridge which spans the Menai Straits were raised by means of an hydraulic press. The cylinder of this machine, the largest which has ever been constructed, was nine feet long, and twenty-two inches in internal diameter; it was capable of raising a weight of two thousand tons.

The principle of the hydraulic press is advantageously employed in cases in which great power is only required at intervals, such as in opening dock gates, in lifts in hotels, warehouses, and the like. In these cases an *accumulator* is used. The piston P is loaded with very great weights, and water is forced into the cylinder B by powerful pumps. From the bottom of this cylinder a tube conducts water to any place where the power is to be applied, and the flow of even small quantities of water can perform a great amount of work.

Suppose, for instance, the area of the piston P is four square feet, and that it has a load of 100 tons; that represents a pressure of over 370 pounds on the square inch, or more than 25 atmospheres. When the large piston sinks through the $\frac{1}{15}$th of an inch about a pint of water will flow out, and this represents a work of about 1,100 foot-pounds.

110. **Water level.**—The *water level* is an application of the conditions

Fig. 79.

of equilibrium in communicating vessels. It consists of a metal tube bent at both ends, in which are fitted glass tubes D and E (fig. 79). It is placed

on a tripod, and water poured in until it rises in both legs. When the liquid is at rest, the level of the water in both tubes is the same; that is, they are both in the same horizontal plane.

This instrument is used in levelling, or ascertaining how much one point is higher than another. If, for example, it is desired to find the difference between the heights of B and A, a *levelling-staff* is fixed on the latter place. This staff consists of a rule formed of two sliding pieces of wood, and supporting a piece of tin plate M, in the centre of which there is a mark. This staff being held vertically at A, an observer looks at it through the level along the surfaces D and E, and directs the holder to raise or lower the slide until the mark is in the prolongation of the line DE. The height AM is then measured, and subtracting it from the height of the level, the height of the point A above B is obtained.

111. **Spirit level.**—The *spirit level* is both more delicate and more accurate than the water level. It consists of a glass tube, AB (fig. 80), very slightly curved; that is, the tube, instead of being a true cylinder as it seems to be, is in fact slightly curved in such a manner that its axis is an arc of a circle of very large radius. It is filled with spirit with the exception of a bubble of air, which tends to occupy the highest part. The tube is placed in a brass case, CD (fig. 81), which is so arranged that when it is in a perfectly horizontal position the bubble of air is exactly between the two points marked in the case.

Fig. 80.

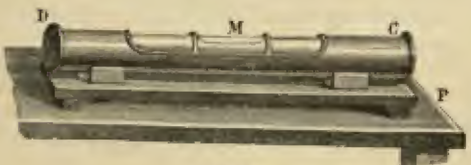

Fig. 81.

To take levels with this apparatus, it is fixed on a telescope, which can consequently be placed in a horizontal position.

112. **Artesian wells.**—All natural collections of water exemplify the tendency of water to find its level. Thus, a group of lakes, such as the great lakes of North America, may be regarded as a number of vessels in communication, and consequently the waters tend to maintain the same level in all. This, too, is the case with the source of a river and the sea, and, as the latter is on the lower level, the river continually flows down to the sea along its bed, which is, in fact, the means of communication between the two.

Perhaps the most striking instance of this class of natural phenomena is that of *artesian wells*. These wells derive their name from the province of Artois, where it has long been customary to dig them, and from whence their use in other parts of France and Europe was derived. It seems, however, that at a very remote period wells of the same kind were dug in China and Egypt.

To understand the theory of these wells, it must be premised that the strata composing the earth's crust are of two kinds: the one *permeable* to water, such as sand, gravel, &c.; the other *impermeable*, such as clay. Let

us suppose, then, a geographical basin of greater or less extent, in which the two impermeable layers AB, CD (fig. 82), enclose between them a permeable layer KK. The rain-water falling on the part of this layer which comes to the surface, which is called the *outcrop*, will filter through it, and following the natural fall of the ground will collect in the hollow of the basin, whence it cannot escape owing to the impermeable strata above and below it. If, now, a vertical hole, I, be sunk down to the water-bearing stratum, the water striving to regain its level will spout out to a height which depends on the difference between the levels of the outcrop and of the point at which the perforation is made.

The waters which feed artesian wells often come from a distance of sixty or seventy miles. The depth varies in different places. The well at

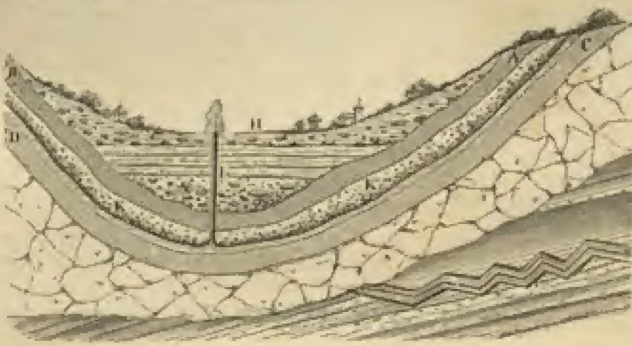

Fig. 82.

Grenelle is 1,800 feet deep; it gives 656 gallons of water in a minute, and is one of the deepest and most abundant which has been made. The temperature of the water is 27° C. t follows from the law of the increase of temperature with the increasing depth below the surface of the ground, that, if this well were 210 feet deeper, the water would have all the year round a temperature of 32° C.; that is, the ordinary temperature of baths.

BODIES IMMERSED IN LIQUIDS.

113. **Pressure supported by a body immersed in a liquid.**—When a solid is immersed in a liquid, every portion of its surface is submitted to a perpendicular pressure which increases with the depth. If we imagine all these pressures decomposed into horizontal and vertical pressures, the first set are in equilibrium. The vertical pressures are obviously unequal, and will tend to move the body upwards.

Let us imagine a cube immersed in a mass of water (fig. 83), and that four of its edges are vertical. The pressures upon the four vertical faces being clearly in equilibrium, we need only consider the pressures exerted on the horizontal faces A and B. The first is pressed downwards by a column of water, whose base is the face A, and whose height is AD, the lower face B

is pressed upwards by the weight of a column of water whose base is the face itself, and whose height is BD (101). The cube, therefore, is urged upwards by a force equal to the difference between these two pressures, which latter is manifestly equal to the weight of a column of water having the same base and the same height as this cube. *Consequently this upward pressure is equal to the weight of the volume of water displaced by the immersed body.*

Fig. 83.

We shall readily see from the following reasoning that every body immersed in a liquid is pressed upwards by a force equal to the weight of the displaced liquid. In a liquid at rest, let us suppose a portion of it of any given shape, regular or irregular, to become solidified, without either increase or decrease of volume. The liquid thus solidified will remain at rest, and therefore must be acted upon by a force equal to its weight, and acting vertically upwards through its centre of gravity; for otherwise motion would ensue. If in the place of the solidified water we imagine a solid of another substance of exactly the same volume and shape, it will necessarily receive the same pressures from the surrounding liquid as the solidified portion did; hence, like the latter, it will sustain the pressure of a force acting vertically upwards through the centre of gravity of the displaced liquid, and equal to the weight of the displaced liquid. If, as almost invariably happens, the liquid is of uniform density, the centre of gravity of the displaced liquid means the centre of gravity of the immersed part of the body *supposed to be of uniform density.* This distinction is sometimes of importance; for example, if a sphere is composed of a hemisphere of iron and another of wood, its centre of gravity would not coincide with its geometrical centre; but if it were placed under water, the centre of gravity of the displaced water would be at the geometrical centre; that is, would have the same position as the centre of gravity of the sphere if of uniform density.

114. **Principle of Archimedes.**—The preceding principles prove that every body immersed in a liquid is submitted to the action of two forces: gravity which tends to lower it, and the buoyancy of the liquid which tends to raise it with a force equal to the weight of the liquid displaced. The weight of the body is either totally or partially overcome by this buoyancy, from which it is concluded that *a body immersed in a liquid loses a part of its weight equal to the weight of the displaced liquid.*

This principle, which is the basis of the theory of immersed and floating bodies, is called the principle of Archimedes, after the discoverer. It may be shown experimentally by means of the *hydrostatic balance* (fig. 84). This is an ordinary balance, each pan of which is provided with a hook; the beam can be raised by means of a toothed rack, which is worked by a little pinion, C. A catch, D, holds the rack when it has been raised. The beam being raised, a hollow brass cylinder, A, is suspended to one of the pans, and below this a solid cylinder, B, whose volume is exactly equal to the capacity of the first cylinder; lastly, an equipoise is placed in the other pan. If now the hollow cylinder A be filled with water the equilibrium is disturbed;

but if at the same time the beam is lowered so that the solid cylinder B becomes immersed in a vessel of water placed beneath it, the equilibrium will be restored. By being immersed in water the cylinder B loses a portion of its weight equal to that of the water in the cylinder A. Now as the capacity of the cylinder A is exactly equal to the volume of the cylinder B, the principle which has been before laid down is proved.

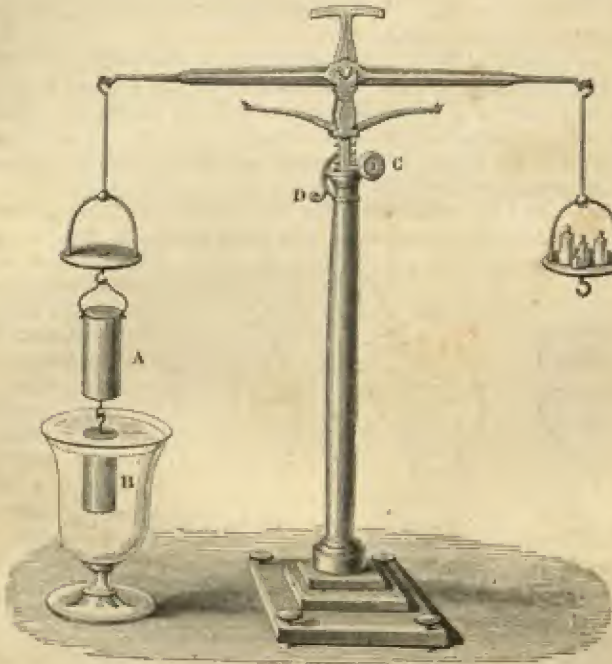

Fig. 84.

115. **Determination of the volume of a body.**—The principle of Archimedes furnishes a method for obtaining the volume of a body of any shape, provided it is not soluble in water. The body is suspended by a fine thread to the hydrostatic balance, and is weighed first in the air, and then in distilled water at 4° C. The loss of weight is the weight of the displaced water, from which the volume of the displaced water is readily calculated. But this volume is manifestly that of the body itself. Suppose, for example, 155 grammes is the loss of weight. This is consequently the weight of the displaced water. Now it is known that a gramme is the weight of a cubic centimetre of water at 4°; consequently, the volume of the body immersed is 155 cubic centimetres.

116. **Equilibrium of floating bodies.**—A body when floating is acted on by two forces, namely its weight, which acts vertically downwards

through its centre of gravity, and the resultant of the fluid pressures, which (113) acts vertically upwards through the centre of gravity of the fluid displaced; but if the body is at rest these two forces must be equal and act in opposite directions; whence follow the conditions of equilibrium, namely :—

i. *The floating body must displace a volume of liquid whose weight equals that of the body.*

ii. *The centre of gravity of the floating body must be in the same vertical line with that of the fluid displaced.*

Thus in fig. 85, if C is the centre of gravity of the body and G that of the displaced fluid, the line GC must be vertical, since when it is so the weight of the body and the fluid pressure will act in opposite directions along the same line, and will be in equilibrium if equal. It is convenient, with reference to the subject of the present article, to speak of the line CG produced as the axis of the body.

Next let it be inquired whether the equilibrium be stable or unstable. Suppose the body to be turned through a small angle (fig. 86), so that the axis takes a position inclined to the vertical. The centre of gravity of the displaced fluid will no longer be G, but some other point, G′. And since the fluid pressure acts vertically upwards through G′, its direction will cut the axis in some point M′, which will generally have different positions according as the inclination of the axis to the vertical is greater or smaller. If the angle is indefinitely small, M′ will have a definite position M, which always admits of determination, and is called the *metacentre.*

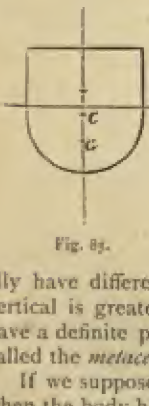

Fig. 85.

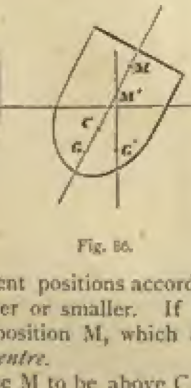

Fig. 86.

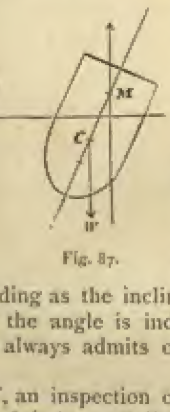

Fig. 87.

If we suppose M to be above C, an inspection of fig. 87 will show that when the body has received an indefinitely small displacement, the weight of the body W, and the resultant of the fluid pressures R tend to bring the body back to its original position; that is, in this case the equilibrium is stable (71). If, on the contrary, M is below C, the forces tend to cause the axis to deviate farther from the vertical, and the equilibrium is unstable. Hence the rule,

iii. *The equilibrium of a floating body is stable or unstable according as the metacentre is above or below the centre of gravity.*

The determination of the metacentre can rarely be effected except by means of a somewhat difficult mathematical process. When, however, the form of the immersed part of a body is spherical it can be readily determined, for since the fluid pressure at each point converges to the centre, and continues to do so when the body is slightly displaced, their resultant must in all cases pass through the centre, which is therefore the metacentre. To illustrate this: let a spherical body float on the surface of a liquid (fig. 88),

then, its centre of gravity and the metacentre both coinciding with the geometrical centre C, its equilibrium is neutral (71). Now suppose a small heavy body to be fastened at P, the summit of the vertical diameter. The centre of gravity will now be at some point G above C. Consequently, the equilibrium is unstable, and the sphere, left to itself, will instantly turn over and will rest when P is the lower end of a vertical diameter.

Fig. 88.

On investigating the position of the metacentre of a cylinder, it is found that when the ratio of the radius to the height is greater than a certain quantity, the position of stable equilibrium is that in which the axis is vertical; but if it be less than that quantity, the equilibrium is stable when the axis is horizontal. For this reason the stump of a tree floats lengthwise, but a thin disc of wood floats flat on the water.

Hence, also, if it is required to make a cylinder of moderate length float with its axis vertical, it is necessary to load it at the lower end. By so doing its centre of gravity is brought below the metacentre.

The determination of the metacentre and of the centre of gravity is of great importance in the stowage of vessels, for on their relative positions the stability depends.

117. **Cartesian diver.**—The different effects of suspension, immersion, and floating are reproduced by means of a well-known hydrostatic toy, the *Cartesian diver* (fig. 89). It consists of a glass cylinder nearly full of water, on the top of which a brass cap, provided with a piston, is hermetically fitted. In the liquid there is a little porcelain figure attached to a hollow glass ball *a*, which contains air and water, and floats on the surface. In the lower part of this ball there is a little hole by which water can enter or escape, according as the air in the interior is more or less compressed. The quantity of water in the globe is such that very little more is required to make it sink. If the piston be slightly lowered, the air is compressed, and this pressure is transmitted to the water of the vessel and the air in the bulb. The consequence is, that a small quantity of water penetrates into the bulb, which therefore becomes heavier and sinks. If the pressure is relieved, the air in the bulb expands, expels the excess of water which had entered it, and the apparatus, being now lighter, rises to the surface. The experiment may also be made by replacing the brass cap and piston by a cover of sheet india-rubber, which is tightly tied over the mouth; when this is pressed by the hand the same effects are produced.

Fig. 89.

118. **Swimming-bladder of fishes.**—Most fishes have an air-bladder below the spine, which is called the *swimming-bladder*. The fish can compress or dilate this at pleasure by means of a muscular effort, and produce the same effects as those just described—that is, it can either rise or sink in water.

119. **Swimming.**—The human body is lighter, on the whole, than an equal volume of water : it consequently floats on the surface, and still better in sea-water, which is heavier than fresh water. The difficulty in swimming consists not so much in floating, as in keeping the head above water, so as to breathe freely. In man the head is heavier than the lower parts, and consequently tends to sink, and hence swimming is an art which requires to be learned. With quadrupeds, on the contrary, the head being less heavy than the posterior parts of the body, remains above water without any effort, and these animals therefore swim naturally.

SPECIFIC GRAVITY—HYDROMETERS.

120. **Determination of specific gravities.**—It has been already explained (24) that the specific gravity of a body, whether solid or liquid, is the number which expresses the relation of the weight of a given volume of this body to the weight of the same volume of distilled water at a temperature of 4°. In order, therefore, to calculate the specific gravity of a body, it is sufficient to determine its weight and that of an equal volume of water, and then to divide the first weight by the second : the quotient is the specific gravity of the body.

Three methods are commonly used in determining the specific gravities of solids and liquids. These are, 1st, the method of the hydrostatic balance ; 2nd, that of the hydrometer ; and 3rd, the specific gravity flask. All three, however, depend on the same principle—that of first ascertaining the weight of a body, and then that of an equal volume of water. We shall first apply these methods to determining the specific gravity of solids, and then to the specific gravity of liquids.

121. **Specific gravity of solids.**—i. *Hydrostatic balance.*—To obtain the specific gravity of a solid by the hydrostatic balance (fig. 84), it is first weighed in the air, and is then suspended to the hook of the balance and weighed in water (fig. 90). The loss of weight which it experiences is, according to Archimedes' principle, the weight of a volume of water equal to its own volume ; consequently, dividing the weight in air by the loss of weight in water, the quotient is the specific gravity required. If P is the weight of the body in air, P′ its weight in water, and D its specific gravity, P – P′ being the weight of the displaced water, we have $D = \frac{P}{P - P'}$.

It may be observed that though the weighing is performed in air, yet, strictly speaking, the quantity required is the weight of the body *in vacuo* ; and when great accuracy is required, it is necessary to apply to the observed weights a correction for the weights of the unequal volumes of air displaced by the substance, and the weights in the other scale pan. The water in which bodies are weighed is supposed to be distilled water at the standard temperature.

ii. *Nicholson's hydrometer.*—The apparatus consists of a hollow metal cylinder B (fig 91), to which is fixed a cone C, loaded with lead. The object of the latter is to bring the centre of gravity below the metacentre, so that the cylinder may float with its axis vertical. At the top is a stem, terminated by a pan, in which is placed the substance whose specific gravity is to be determined. On the stem a standard point, *o*, is marked.

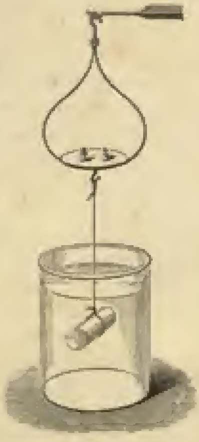

Fig. 90.

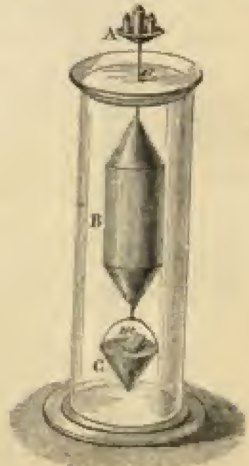

Fig. 91.

The apparatus stands partly out of the water, and the first step is to ascertain the weight which must be placed in the pan in order to make the hydrometer sink to the standard point *o*. Let this weight be 125 grains, and let sulphur be the substance whose specific gravity is to be determined. The weights are then removed from the pan, and replaced by a piece of sulphur which weighs less than 125 grains, and weights added until the hydrometer is again depressed to the standard *o*. If, for instance, it has been necessary to add 55 grains, the weight of the sulphur is evidently the difference between 125 and 55 grains; that is, 70 grains. Having thus determined the weight of the sulphur in air, it is now only necessary to ascertain the weight of an equal volume of water. To do this, the piece of sulphur is placed in the lower pan C at *m*, as represented in the figure. The whole weight is not changed, nevertheless the hydrometer no longer sinks to the standard; the sulphur, by immersion, has lost a part of its weight equal to that of the water displaced. Weights are added to the upper pan until the hydrometer sinks again to the standard. This weight, 34·4 grains, for example, represents the weight of the volume of water displaced; that is, of the volume of water equal to the volume of the sulphur. It is only necessary, therefore, to divide 70 grains, the weight in air, by 34·4 grains, and the quotient 2·03 is the specific gravity.

If the body in question is lighter than water it tends to rise to the surface, and will not remain on the lower pan C. To obviate this, a small movable cage of fine wire is adjusted so as to prevent the ascent of the body. The experiment is in other respects the same.

122. **Specific gravity bottle. Pyknometer.**—When the specific gravity of a substance in a state of powder is required, it can be found most conveniently by means of the *pyknometer*, or specific gravity bottle. This instrument is a bottle, in the neck of which is fitted a thermometer A, an enlargement on the stem being carefully ground for this purpose (fig. 92). In the

side is a narrow capillary stem widened at the top and provided with a stopper, as shown in the figure. On this tube is a mark *m*, and the thermometer stopper having been inserted, at each weighing the bottle is filled with water exactly to this mark. The bottle may conveniently have dimensions such that when the thermometer stopper is inserted and the liquid filled to the mark *m*, it represents a definite volume. This is done by filling the bottle when wholly under water, and putting in the stopper while it is immersed. The bottle and the tube are then completely filled, and the quantity of water in excess is removed by blotting paper. To find the specific gravity proceed as follows:—Having weighed the powder, place it in one of the scale pans, and with it the bottle filled exactly to *m*, and carefully dried. Then balance it by placing small shot, or sand, in the other pan. Next, remove the bottle and pour the powder into it, and, as before, fill it up with water to the mark *a*. On replacing the bottle in the scale pan it will no longer balance the shot, since the powder has displaced a volume of water equal to its own volume. Place weights in the scale pan along with the bottle until they balance the shot. These weights give the weight of the water displaced. Then the weight of the powder, and the weight of an equal bulk of water being known, its specific gravity is determined as before. The thermometer gives the temperature at which the determination is made, and thus renders it easy to make a correction (125).

Fig. 52

It is important in this determination to remove the layer of air which adheres to the powder, and unduly increases the quantity of water expelled. This is effected by placing the bottle under the receiver of an air-pump and exhausting. The same result is obtained by boiling the water in which the powder is placed.

123. **Bodies soluble in water.**—If the body, whose specific gravity is to be determined by any of these methods, is soluble in water, the determination is made in some liquid in which it is not soluble, such as oil of turpentine or naphtha, the specific gravity of which is known. The specific gravity is obtained by multiplying the number obtained in the experiment by the specific gravity of the liquid used for the determination.

Suppose, for example, a determination of the specific gravity of potassium has been made in naphtha. For equal volumes, P represents the weight of the potassium, P′ that of the naphtha, and P″ that of water; consequently

$\frac{P}{P'}$ will be the specific gravity of the substance in reference to naphtha, and $\frac{P'}{P''}$ the specific gravity of the naphtha in reference to water. The product of these two fractions $\frac{P}{P''}$ is the specific gravity of the substance compared with water.

In determining the specific gravity of porous substances, they are varnished before being immersed in water, which renders them impervious to moisture without altering their volume.

Specific gravity of solids at zero as compared with distilled water at 4° C.

Platinum, rolled	22·069	Statuary marble	2·837
" cast	20·337	Aluminium	2·680
Gold, stamped	19·362	Rock crystal	2·653
" cast	19·258	St. Gobin glass	2·488
Lead, cast	11·352	China porcelain	2·38
Silver, cast	10·474	Sèvres porcelain	2·14
Bismuth, cast	9·822	Native sulphur	2·033
Copper, drawn wire	8·878	Ivory	1·917
" cast	8·788	Anthracite	1·800
German silver	8·432	Compact coal	1·329
Brass	8·383	Amber	1·078
Steel, not hammered	7·816	Sodium	0·970
Iron, bar	7·788	Melting ice	0,930
Iron, cast	7·207	Potassium	0·865
Tin, cast	7·291	Beech	0·852
Zinc, cast	6·861	Oak	0·845
Antimony, cast	6·712	Elm	0·800
Iodine	4·950	Yellow Pine	0·657
Heavy spar	4·430	Lithium	0·585
Diamonds	3·531 to 3·501	Common poplar	0·389
Flint glass	3·329	Cork	0·240

In this table the woods are supposed to be in the ordinary air-dried condition.

124. **Specific gravity of liquids.**—i. *Method of the hydrostatic balance.*—From the pan of the hydrostatic balance a body is suspended, on which the liquid, whose specific gravity is to be determined, exerts no chemical action; for example, a ball of platinum. This is then successively weighed in air, in distilled water, and in the liquid. The loss of weight of the body in these two liquids is noted. They represent respectively the weights of equal volumes of water and of the given liquid, and consequently it is only necessary to divide the second of them by the first to obtain the required specific gravity.

Let P be the weight of the platinum ball in air, P′ its weight in water, P″ its weight in the given liquid, and let D be the specific gravity sought. The weight of the water displaced by the platinum is P – P′, and that of the second liquid is P – P″, from which we get $D = \frac{P - P''}{P - P'}$.

ii. *Fahrenheit's hydrometer.*—This instrument (fig. 93) resembles Nicholson's hydrometer, but it is made of glass, so as to be used in all liquids. At

its lower extremity, instead of a pan, it is loaded with a small bulb containing mercury. There is a standard mark on the stem.

The weight of the instrument is first accurately determined in air; it is then placed in water, and weights added to the scale pan until the mark on the stem is level with the water. It follows, from the first principle of the equilibrium of floating bodies, that the weight of the hydrometer, together with the weight in the scale pan, is equal to the weight of the volume of the displaced water. In the same manner, the weight of an equal volume of the given liquid is determined, and the specific gravity is found by dividing the latter weight by the former.

Fig. 93.

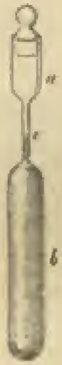

Fig. 94.

Neither Fahrenheit's nor Nicholson's hydrometers give such accurate results as the hydrostatic balance.

iii. *Specific gravity bottle.*—This has been already described (122). In determining the specific gravity of a liquid, a bottle of special construction is used; it consists of a cylindrical reservoir *b* (fig. 94), to which is fused a capillary tube *c*, and to this again a wider tube *a* closed with a stopper. The bottle is first weighed empty, and then successively full of water to the mark *c* on the capillary stem and of the given liquid. If the weight of the bottle be subtracted from the two weights thus obtained, the result represents the weights of equal volumes of the liquid, and of water, from which the specific gravity is obtained by division.

125. **On the observation of temperature in ascertaining specific gravities.**—As the volume of a body increases with the temperature, and as this increase varies with different substances, the specific gravity of any given body is not exactly the same at different temperatures; and, consequently, a certain fixed temperature is chosen for those determinations. That of water, for example, has been made at 4° C., for at this point it has the greatest density. The specific gravities of other bodies are assumed to be taken at zero; but, as this is not always possible, certain corrections must be made, which we shall consider in the Book on Heat.

Specific gravities of liquids at zero, compared with that of water at 4° C. as unity.

Mercury	13·598	Sea-water	1·026
Bromine	2·960	Distilled water at 4° C.	1·000
Sulphuric acid	1·841	„ „ at 0° C.	0·999
Chloroform	1·525	Claret	0·994
Nitric acid	1·420	Olive oil	0·915
Bisulphide of carbon	1·293	Oil of turpentine	0·870
Glycerine	1·260	Oil of lemon	0·852
Hydrochloric acid	1·[illegible]	Petroleum	0·836
Blood	1·060	Absolute alcohol	0·793
Milk	1·032	Ether	0·713

126. **Use of tables of specific gravity.**—Tables of specific gravity

admit of numerous applications. In mineralogy the specific gravity of a mineral is often a highly distinctive character. By means of tables of specific gravities the weight of a body may be calculated when its volume is known, and conversely the volume when its weight is known.

With a view to explaining the last-mentioned use of these tables, it will be well to premise a statement of the connection existing between the British units of length, capacity, and weight. It will manifestly be sufficient for this purpose to define that which exists between the yard, gallon, and pound avoirdupois, since other measures stand to these in well-known relations. The *yard*, consisting of 36 inches, may be regarded as the primary unit. Though it is essentially an arbitrary standard, it is determined by this, that the simple pendulum which makes one oscillation in a mean second, at London on the sea-level, is 39·13983 inches long. The *gallon* contains 277·274 cubic inches. A gallon of distilled water at the standard temperature weighs 10 pounds avoirdupois or 70,000 grains troy; or, which comes to the same thing, one cubic inch of water weighs 252·5 grains.

On the French system the *metre* is a primary unit, and is so chosen that 10,000,000 metres are the length of a quadrant of the meridian from either pole to the equator. The metre contains 10 *decimetres*, or 100 *centimetres*, or 1,000 *millimetres*; its length equals 1·0936 yards. The unit of the measure of capacity is the *litre* or cubic decimetre. The unit of weight is the *gramme*, which is the weight of a cubic centimetre of distilled water at 4° C. The *kilogramme* contains 1,000 grammes, or is the weight of a decimetre of distilled water at 4° C. The *gramme* equals 15·443 grains.

If V is the number of cubic centimetres (or decimetres) in a certain quantity of distilled water at 4° C., and P its weight in grammes (or kilogrammes), it is plain that $P = V$. Now consider a substance whose specific gravity is D; every cubic centimetre of this substance will weigh as much as D cubic centimetres of water, and therefore V centimetres of this substance will weigh as much as DV centimetres of water. Hence if P is the weight of the substance in grammes, we have $P = DV$. If, however, V is the volume in cubic inches, and P the weight in grains, we shall have $P = 252{\cdot}5\ DV$.

As an example, we may calculate the internal diameter of a glass tube. Mercury is introduced, and the length and weight of the column at 4° C. are accurately determined. As the column is cylindrical, we have $V = \pi r^2 l$, where r is the radius, and l the length of the column in centimetres. Hence if D is the specific gravity of mercury, and P the weight of the column in grammes, we have $P = \pi r^2 l D$, and therefore

$$r = \sqrt{\frac{P}{\pi D l}}$$

If r and l are in inches and P in grains, we shall have $P = 252{\cdot}5\pi r^2 l D$, and therefore

$$r = \sqrt{\frac{P}{252{\cdot}5\pi D l}}$$

In a similar manner the diameter of very fine metal wires can be determined with great accuracy.

127. **Hydrometers with variable volume.**—The hydrometers of Nicholson and Fahrenheit are called *hydrometers of constant volume, but variable weight*, because they are always immersed to the same extent, but carry

different weights. There are also *hydrometers of variable volume but of constant weight.* These instruments, known under the different names of *acidometer*, *alcoholometer*, *lactometer*, and *saccharometer*, are not used to determine the exact specific gravity of the liquids, but to show whether the acids, alcohols, milk, solutions of sugar, &c., under investigation, are more or less concentrated.

128. **Beaumé's hydrometer.**—This, which was the first of these instruments, may serve as a type of them. It consists of a glass tube (fig. 95) loaded at the bottom with mercury, and with a bulb blown in the middle. The stem, the external diameter of which is as regular as possible, is hollow, and the scale is marked upon it.

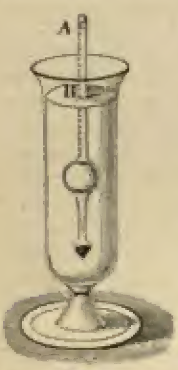

Fig. 95.

The graduation of the instrument differs according as the liquid, for which it is to be used, is heavier or lighter than water. In the first case, it is so constructed that it sinks in water nearly to the top of the stem, to a point A, which is marked zero. A solution of fifteen parts of salt in eighty-five parts of water is made, and the instruments immersed in it. It sinks to a certain point on the stem, B, which is marked 15; the distance between A and B is divided into 15 equal parts, and the graduation continued to the bottom of the stem. Sometimes the graduation is on a piece of paper inside the stem.

The hydrometer thus graduated only serves for liquids of a greater specific gravity than water, such as acids and saline solutions. For liquids lighter than water a different plan must be adopted. Beaumé took for zero the point to which the apparatus sank in a solution of 10 parts of salt in 90 of water, and for 10° he took the level in distilled water. This distance he divided into 10°, and continued the division to the top of the scale.

The graduation of these hydrometers is entirely conventional, and they give neither the densities of the liquids nor the quantities dissolved. But they are very useful in making mixtures or solutions in given proportions, the results they give being sufficiently near in the majority of cases. For instance, it is found that a well-made syrup marks 35 on Beaumé's hydrometer, from which a manufacturer can readily judge whether a syrup which is being evaporated has reached the proper degree of concentration.

129. **Gay-Lussac's alcoholometer.**—This instrument is used to determine the strength of spirituous liquors; that is, the proportion of pure alcohol which they contain. It differs from Beaumé's hydrometer in the graduation.

Mixtures of absolute alcohol and distilled water are made containing 5, 10, 20, 30, &c., per cent. of the former. The alcoholometer is so constructed that, when placed in pure distilled water, the bottom of its stem is level with the water, and this point is zero. It is next placed in absolute alcohol, which marks 100°, and then successively in mixtures of different strengths, containing 10, 20, 30, &c., per cent. The divisions thus obtained are not exactly equal, but their difference is not great, and they are subdivided into ten divisions, each of which marks *one* per cent. of absolute alcohol in a liquid. Thus a brandy in which the alcoholometer stood at 48° would contain 48 per cent. of absolute alcohol, and the rest would be water.

All these determinations are made at 15° C., and for that temperature only are the indications correct. For, other things being the same, if the temperature rises, the liquid expands, and the alcoholometer will sink, and the contrary if the temperature fall. To obviate this error, Gay-Lussac constructed a table which for each percentage of alcohol gives the reading of the instrument for each degree of temperature from 0° up to 30°. When the exact analysis of an alcoholic mixture is to be made, the temperature of the liquid is first determined, and then the point to which the alcoholometer sinks in it. The number in the table corresponding to these data indicates the percentage of alcohol. From its giving the percentage of alcohol, this is often called the *centesimal alcoholometer.*

130. **Salimeters.**—*Salimeters*, or instruments for indicating the percentage of salt contained in a solution, are made on the principle of the centesimal alcoholometer. They are graduated by immersing them in pure water which gives the zero, and then in solutions containing different percentages, 5, 10, 20, &c., of the salt, and marking on the scale the corresponding points. These instruments are open to the objection that every salt requires a special instrument. Thus one graduated for common salt would give totally false indications in a solution of nitre.

Lactometers and *vinometers* are similar instruments, and are used for measuring the quantity of water which is introduced into milk or wine for the purpose of adulteration. But their use is limited, because the density of these liquids is very variable, even when they are perfectly natural, and an apparent fraud may be really due to a bad natural quality of wine or of milk. *Urinometers*, which are of extensive use in medicine, are based on the same principle.

131. **Densimeter.**—The *densimeter* is an apparatus for indicating the specific gravity of a liquid. Rosseau's densimeter (fig. 96) is of great use, in many scientific investigations, in determining the specific gravity of a small quantity of a liquid. It has the same form as Beaumé's hydrometer, but on the upper part of the stem there is a small tube AC, in which is placed the substance to be determined. A mark A on the side of the tube indicates a measure of a cubic centimetre.

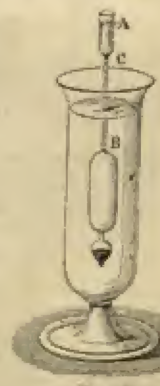

Fig. 96.

The instrument is so constructed that when AC is empty it sinks in distilled water to a point, B, just at the bottom of the stem. It is then filled with distilled water to the height measured on the tube AC, which indicates a cubic centimetre, and the point to which it now sinks is 20°. The interval between 0 and 20 is divided into 20 equal parts, and this graduation is continued to the top of the scale. As this is of uniform bore, each division corresponds to $\frac{1}{20}$ gramme or 0·05.

To obtain the density of any liquid, bile for example, the tube is filled with it up to the mark A; if the densimeter sinks to $20\frac{1}{2}$ divisions, its weight is $0·05 \times 20·5 = 1·025$: that is to say, that with equal volumes, the weight of water being 1, that of bile is 1·025. The specific gravity of bile is therefore 1·025.

CHAPTER II.

CAPILLARITY, ENDOSMOSE, EFFUSION, ABSORPTION, AND IMBIBITION.

132. **Capillary phenomena.**—When solid bodies are placed in contact with liquids, a class of phenomena is produced called *capillary phenomena*, because they are best seen in tubes whose diameters are comparable with the diameter of a hair. These phenomena are treated of in physics under the head of *capillarity* or *capillary attraction*; the latter expression is also applied to the force which produces the phenomena.

The phenomena of capillarity are very various, but may all be referred to the mutual attraction of the liquid molecules for each other, and to the attraction between these molecules and solid bodies. The following are some of these phenomena :—

When a body is placed in a liquid which wets it—for example, a glass rod in water—the liquid, as if not subject to the laws of gravitation, is raised upwards against the sides of the solid, and its surface, instead of being horizontal, becomes slightly concave (fig. 97). If, on the contrary, the solid is

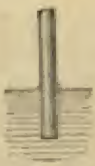
Fig. 97.


Fig. 98.

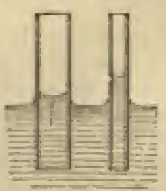
Fig. 99.


Fig. 100.

one which is not moistened by the liquid, as glass by mercury, the liquid is depressed against the sides of the solid, and assumes a convex shape, as represented in fig. 98. The surface of the liquid exhibits the same concavity or convexity against the sides of a vessel in which it is contained, according as the sides are or are not moistened by the liquid.

These phenomena are much more apparent when a tube of small diameter is placed in a liquid. And according as the tubes are or are not moistened by the liquid, an ascent or a depression of the liquid is produced which is greater in proportion as the diameter is less (figs. 99 and 100).

When the tubes are moistened by the liquid, its surface assumes the form of a concave hemispherical segment, called the *concave meniscus* (fig. 99); when the tubes are not moistened, there is a *convex meniscus* (fig. 100).

133. **Laws of the ascent and depression in capillary tubes.**—The most important law in reference to capillarity is known as *Jurin's law*. It

is that the height of the ascent of one and the same liquid in a capillary tube is inversely as the diameter of the tube. Thus, if water rises to a height of 30 mm. in a tube 1 mm. in diameter, it will only rise to a height of 15 mm. in a tube 2 mm. in diameter, but to a height of 300 mm. in a tube 0·1 mm. in diameter. This law has been verified with tubes whose diameters ranged from 5 mm. to 0·07 mm. It presupposes that the liquid has previously moistened the tube.

The height to which a liquid rises in a tube, diminishes as the temperature rises. Thus in a capillary tube in which water stood at a height of 30·7 mm. at 0°, it stood at 28·6 mm. at 35°, and at 26 mm. at 80°.

Provided the liquid moistens the tube, neither its thickness nor its nature has any influence on the height to which the liquid rises. Thus water rises to the same height in tubes of different kinds of glass and of rock crystal, provided the diameters are the same.

The nature of the liquid is of great importance; of all liquids water rises the highest; thus in a glass tube 1·29 mm. in diameter, the heights of water, alcohol, and turpentine were respectively 23·16, 9·18, and 9·85 millimetres.

In regard to the depression of liquids in tubes which they do not moisten, Jurin's law has not been found to hold with the same accuracy. The reason for this is probably to be found in the following circumstances:—When a liquid moistens a capillary tube, a very thin layer of liquid is formed against the sides, and remains adherent even when the liquid sinks in the tube. The ascent of the column of liquid takes place then, as it were, inside a central tube, with which it is physically and chemically identical. The ascent of the tube is thus an act of cohesion. It is therefore easy to understand why the nature of the sides of the capillary tube should be without influence on the height of the ascent, which only depends on the diameter.

With liquids, on the contrary, which do not moisten the sides of the tube, the capillary action takes place between the sides and the liquid. The nature and structure of the sides are never quite homogeneous, and there is always, moreover, a layer of air on the inside, which is not dissolved by the liquid. These two causes exert undoubtedly a disturbing influence on the law of Jurin.

134. **Ascent and depression between parallel or inclined surfaces.**—When two bodies of any given shape are dipped in water, analogous capillary phenomena are produced, provided the bodies are sufficiently near. If, for example, two parallel glass plates are immersed in water at a very small distance from each other, water will rise between the two plates in the inverse ratio of the distance which separates them. The height of the ascent for any given distance is half what it would be in a tube whose diameter is equal to the distance between the plates.

If the parallel plates are immersed in mercury, a corresponding depression is produced, subject to the same laws.

If two glass plates AB and AC with their planes vertical and inclined to one another at a small angle, as represented in fig. 101, have their ends dipped into a liquid which wets them, the liquid will rise between them. The elevation will be greatest at the line of contact of the plates and from thence gradually less, the surface taking the form of an equilateral hyper-

bola, whose asymptotes are respectively the line of intersection of the plates, and the line in which the plates cut the horizontal surface of the liquid.

If a drop of water be placed within a conical glass tube whose angle is small and axis horizontal, it will have a concave meniscus at each end

Fig. 101.

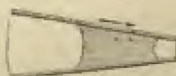

Fig. 102.

Fig. 103.

(fig. 102), and will tend to move towards the vertex. But if the drop be of mercury it will have a convex meniscus at each end (fig. 103), and will tend to move from the vertex.

135. **Attraction and repulsion produced by capillarity.**—The attractions and repulsions observed between bodies floating on the surface of liquids are due to capillarity, and are subject to the following laws :—

i. When two floating balls both moistened by the liquid—for example, cork upon water—are so near that the liquid surface between them is not level, an attraction takes place.

ii. The same effect is produced when neither of the balls is moistened, as is the case with balls of wax on water.

iii. Lastly, if one of the balls is moistened and the other not, as a ball of cork and a ball of wax in water, they repel each other if the curved surfaces of the liquid in their respective neighbourhoods intersect.

As all these capillary phenomena depend on the concave or convex curvature which the liquid assumes in contact with the solid, a short explanation of the cause which determines the form of this curvature is necessary.

136. **Cause of the curvature of liquid surfaces in contact with solids.**—The form of the surface of a liquid in contact with a solid depends on the relation between the attraction of the solid for the liquid, and of the mutual attraction between the molecules of the liquid.

Let m be a liquid molecule (fig. 104) in contact with a solid. This molecule is acted upon by three forces: by gravity which attracts it in the direction of the vertical mP; by the attraction of the liquid F, which acts in the direction mF; and by the attraction of the plate n, which is exerted in the direction mn. According to the relative intensities of these forces, their resultant can take three positions :—

i. The resultant is in the direction of the vertical mR (fig. 104). In this case the surface m is plane and horizontal; for, from the condition of the equilibrium of liquids, the surface must be perpendicular to the force which acts upon the molecules.

ii. If the force n increases or F diminishes, the resultant R is within the

angle *nm*P (fig. 105); in this case the surface takes a direction perpendicular to *m*R, and becomes concave.

iii. If the force F increases, or *n* diminishes, the resultant R takes the

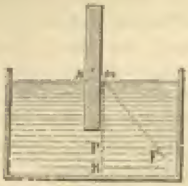
Fig. 104.

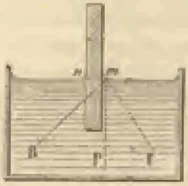
Fig. 105.

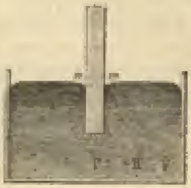
Fig. 106.

direction *m*R (fig. 106) within the angle P*m*F, and the surface, becoming perpendicular to this direction, is convex.

137. **Influence of the curvature on capillary phenomena.**—The elevation or depression of a liquid in a capillary tube depends on the concavity or convexity of the meniscus. In a concave meniscus, *abcd* (fig 107), the liquid molecules are sustained in equilibrium by the forces acting on them, and

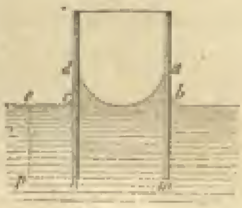
Fig. 107.

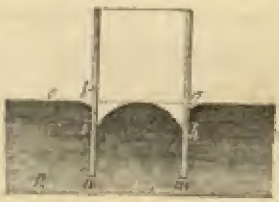
Fig. 108.

they exercise no downward pressure on the inferior layers. On the contrary, in virtue of the molecular attraction, they act on the nearest inferior layers, from which it follows that the pressure on any layer, *mn*, in the interior of the tube, is less than if there were no meniscus. The consequence is, that the liquid ought to rise in the tube until the internal pressure on the layer *mn* is equal to the pressure, *op*, which acts externally on a point, *p*, of the same layer.

Where the meniscus is convex (fig. 108), equilibrium exists in virtue of the molecular forces acting on the liquid; but as the molecules which would occupy the same space *ghik*, if there were no molecular action, do not exist, they exercise no attraction on the lower layers. Consequently, the pressure on any layer *mn*, in the interior of the tube, is greater than if the space *ghik* were filled, for the molecular forces are more powerful than gravity. The liquid ought therefore to sink in the tube until the internal pressure on a layer, *mn*, is equal to the external pressure on any point, *p*, of this layer.

138. **Tension of the free surface of liquids.**—The *free surface* of a liquid is that which is bounded by a gas or by vacuum; it has greater cohesion than any layer of the liquid in the interior. For consider any particle at the surface, it will be attracted by the adjacent particles in all directions except in that above the surface. The attractions acting laterally will compensate each other; and as there are no attractions exerted by the particles

of the liquid above the surface to counteract those acting from the interior, the latter will exercise a considerable pull towards the interior. The effect of this is to lessen the mobility of particles on the surface, while those in the interior are quite mobile; the surface, as it were, is stretched by an elastic skin, the effect being the same as if the surface layer exerted a pressure on the interior. This *surface tension*, as it may be called, is greater, the greater the cohesion of the liquid.

When the surface of a liquid increases, more particles enter into the condition of the surface layer, to effect which a certain amount of work is required. On the other hand, when the surface is diminished, the molecules pass into the state of the internal layer, and they perform work. The work done when a square mm. of surface passes into the interior is called the *coefficient of surface tension.*

The surface tension depends on the form of the surface. It has been determined in the case of spheroidal bodies. If the pressure which is exerted on a *plane* surface be called P, the pressure p, on a spherical surface of radius ρ, is $p = P + \frac{2\phi}{\rho}$ for convex, and $p = P - \frac{2\phi}{\rho}$ for concave surfaces.

Hence for a spheroidal shell, the internal radius OA of which is ρ, and its thickness $AB = d$, the pressure of the outer layer is $p = P + \frac{2\phi}{\rho + d}$, and of the inner layer $p_1 = P - \frac{2\phi}{\rho}$, and the resultant is their difference $= \frac{2\phi}{\rho + d} + \frac{2\phi}{\rho}$; a pressure exerted inwards, since $p > p_1$. This is well illustrated by blowing a soap-bubble on a glass tube. So long as the other end of the tube is closed, the bubble remains, the elastic force of the enclosed air counterbalancing the tension of the surface; but when the tube is opened, the tension of the surface being unchecked, the bubble gradually contracts and finally disappears.

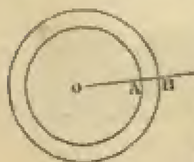

Fig. 109.

Insects can often move on the surface of water, without sinking. This phenomenon is caused by the fact that, as their feet are not wetted by the water, a depression is produced, and the elastic reaction of the surface layer keeps them up in spite of their weight. Similarly a sewing needle, gently placed on water, does not sink, because its surface, being covered with an oily layer, does not become wetted. The pressure of the needle brings about a concavity, the surface tension of which acts in opposition to the weight of the needle. But if washed in alcohol or in potash, it at once sinks to the bottom.

A drop of mercury on a table has a spherical shape, which, like that of the heavenly bodies, is due to attraction. The globule of mercury behaves as if its molecules had no weight, since it remains spherical. That is, the molecular attraction is far greater than the weight, which only alters the shape of the globule if the quantity of mercury is much greater; it then flattens, but always retains at its edge the convex form which attraction imparts to it.

139. **Various capillary phenomena.**—The following facts are among the many which are caused by capillarity:—

When a capillary tube is immersed in a liquid which moistens it, and is then carefully removed, the column of liquid in the tube is seen to be longer than while the tube was immersed in the liquid. This arises from the fact that a drop adheres to the lower extremity of the tube and forms a concave meniscus, which concurs with that of the upper meniscus to form a longer column (132).

For the same reason a liquid does not overflow in a capillary tube, although the latter may be shorter than the liquid column which would otherwise be formed in it. For when the liquid reaches the top of the tube, its upper surface, though previously concave, becomes convex, and, as the downward pressure becomes greater than if the surface were plane, the ascending motion ceases.

It is from capillarity that oil ascends in the wicks of lamps, that water rises in woods, sponge, bibulous paper, sugar, sand, and in all bodies which possess pores of a perceptible size. In the cells of plants the sap rises with great force, for here we have to do with vessels whose diameter is less than 0·01 mm. Efflorescence of salts is also due to capillarity; a solution rising against the side of a vessel, the water evaporates, and the salt forms on the side a means of furthering still more the ascent of a liquid. Capillarity is, moreover, the cause of the following phenomenon:—When a porous substance, such as gypsum, or chalk, or even earth, is placed in a porous vessel of unbaked porcelain, and the whole is dipped in water, the water penetrates into the pores, and the air is driven inwards, so that it is under four or five times its usual pressure and density.

Jamin has proved this by cementing a manometer into blocks of chalk, gypsum, &c., and he has made it probable that a pressure of this kind, exerted upon the roots, promotes the ascent of sap in plants.

ENDOSMOSE, EFFUSION, ABSORPTION, AND IMBIBITION.

140. **Endosmose and exosmose.**—When two different liquids are separated by a thin porous partition, either inorganic or organic, a current sets in from each liquid to the other; to these currents the names *endosmose* and *exosmose* are respectively given. These terms, which signify *impulse from within and impulse from without*, were originally introduced by Dutrochet, who first drew attention to these phenomena. The general phenomenon may be termed *diosmose*. They may be well illustrated by means of the *endosmometer*. This consists of a long tube, at the end of which a membranous bag is firmly bound (fig. 110). The bag is then filled with a strong syrup, or some other solution denser than water, such as milk or albumen, and is immersed in water. The liquid is found gradually to rise in the tube, to a height which may attain several inches; at the same time, the level of the liquid in which the endosmometer is immersed becomes lower. It follows, therefore, that some of the external liquid has passed through the membrane and has mixed with the internal liquid. The external liquid, moreover, is found to contain some of the internal liquid. Hence two currents have been produced in opposite directions. The flow of the liquid towards that which increases in volume is *endosmose*, and the

current in the opposite direction is *exosmose.* If water is placed in the bag, and immersed in the syrup, endosmose is produced from the water towards the syrup, and the liquid in the interior diminishes in volume while the level of the exterior is raised.

Fig. 110.

The height of the ascent in the endosmometer varies with different liquids. Of all vegetable substances, sugar is that which, for the same density, has the greatest power of endosmose, while albumen has the highest power of all animal substances. In general it may be said that endosmose takes place towards the denser liquid. Alcohol and ether form an exception to this; they behave like liquids which are denser than water. With acids, according as they are more or less dilute, the endosmose is from the water towards the acid, or from the acid towards the water.

According to Dutrochet, it is necessary for the production of endosmose: i. that the liquids be different but capable of mixing, as alcohol and water—there is no diosmose, for instance, with water and oil: ii. that the liquids be of different densities: and iii. that the membrane must be permeable to at least one of the substances.

The current through thin inorganic plates is feeble, but continuous, while organic membranes are rapidly decomposed, and diosmose then ceases.

The well-known fact that dilute alcohol kept in a porous vessel becomes concentrated depends on endosmose. If a mixture of alcohol and water be kept for some time in a bladder, the volume diminishes, but the alcohol becomes much more concentrated. The reason, doubtless, is that the bladder permits the diosmose of water rather than that of alcohol.

Dutrochet's method is not adapted for quantitative measurements, for it does not take into account the hydrostatic pressure produced by the column. Jolly has examined the endosmose of various liquids by determining the weights of the bodies diffused. He calls the *endosmotic equivalent* of a substance the number which expresses how many parts by weight of water pass through the bladder in exchange for one part by weight of the substance. The following are some of the endosmotic equivalents which he determined— ;

Sulphuric acid	0·4	Sulphate of copper . . .	9·5
Alcohol	4·2	" magnesium . .	11·7
Chloride of sodium . .	4·3	Caustic potass	215·0
Sugar	7·1		

He also found that the endosmotic equivalent increases with the temperature, and that the quantities of substances which pass in equal times through the bladder are proportional to the strengths of the solutions.

141. **Diffusion of liquids.**—If oil be poured on water no tendency to intermix is observed, and even if the two liquids be violently agitated together, on allowing them to stand, two separate layers are formed. With alcohol and water the case is different; if alcohol, which is specifically lighter, be poured upon water, the liquids gradually intermix, spite of the difference of their specific gravities: they *diffuse* into one another.

This point may be illustrated by the experiment represented in fig. 112. A tall jar contains water coloured by solution of blue litmus; by means of a funnel some dilute sulphuric acid is carefully poured in, so as to form a layer at the bottom; the colour of the solution is changed into red, progressing upwards, and after forty-eight hours the change is complete—a result of

Fig. 111.

Fig. 112.

the action of the acid, and a proof, therefore, that it has diffused throughout the entire mass.

The laws of this diffusion, in which no porous diaphragm is used, have been completely investigated by Graham. The method, by which his latest experiments were made, was the following:—A small wide-necked bottle A (fig. 111) filled with the liquid, whose rate of diffusion was to be examined, was closed by a thin glass disc and placed in a larger vessel B, in which water was poured to a height of about an inch above the top of the bottle. The disc was carefully removed, and then after a given time successive layers were carefully drawn off by means of a siphon or pipette, and their contents examined.

The general results of these investigations may be thus stated:—

i. When solutions of the same substance, but of different strengths, are taken, the quantities diffused in equal times are proportional to the strengths of the solutions.

ii. In the case of solutions containing equal weights of different substances, the quantities diffused vary with the nature of the substances. Saline substances may be divided into a number of *equidiffusive groups*, the rates of diffusion of each group being connected with the others by a simple numerical relation.

iii. The quantity diffused varies with the temperature. Thus, taking the rate of diffusion of hydrochloric acid at 15° C. as unity, at 49° C. it is 2·18.

iv. If two substances which do not combine be mixed in solution, they may be partially separated by diffusion, the more diffusive one passing out most rapidly. In some cases chemical decomposition even may be effected by diffusion. Thus, bisulphate of potassium is decomposed into free sulphuric acid and neutral sulphate of potassium.

v. If liquids be dilute a substance will diffuse into water, containing another substance dissolved, as into pure water; but the rate is materially reduced if a portion of the same diffusing substance be already present.

The following table gives the approximate times of equal diffusion :—

Hydrochloric acid . . .	1·0	Sulphate of magnesium . .	7·0
Chloride of sodium . . .	2·3	Albumen	49·0
Sugar	7·0	Caramel	98·0

It will be seen from the above table that the difference between the rates of diffusion is very great. Thus, msulphate of agnesium, one of the least diffusible saline substances, diffuses 7 times as rapidly as albumen and 14 times as rapidly as caramel. These last substances, like hydrated silicic acid, starch, dextrine, gum, &c., constitute a class of substances which are characterised by their incapacity for taking the crystalline form and by the mucilaginous character of their hydrates. Considering gelatine as the type of this class, Graham has proposed to call them *colloids* (κόλλη, glue), in contradistinction to the far more easily diffusible *crystalloid* substances.

This is possibly owing to the fact that the larger molecules only pass with difficulty through minute apertures.

Graham has proposed a method of separating bodies based on their unequal diffusibility, which he calls *dialysis*. His *dialyser* (fig. 113) consists of

Fig. 113.

Fig. 114.

a ring of gutta percha, over which is stretched while wet a sheet of parchment paper, forming thus a vessel about two inches high and ten inches in diameter, the bottom of which is of parchment paper. After pouring in the mixed solution to be dialysed, the whole is floated on a vessel containing a very large quantity of water (fig. 114). In the course of one or two days a more or less complete separation will have been effected. Thus a solution of arsenious acid mixed with various kinds of food readily diffuses out. The process has received important applications to laboratory and pharmaceutical purposes.

Diosmose plays a most important part in organic life; the cell-walls are diaphragms, through which the liquids in the cells set up diosmotic communications.

142. **Endosmose of gases.**—The phenomena of endosmose are seen in a high degree in the case of gases, the treatment of which we may here anticipate. When two different gases are separated by a porous diaphragm, an interchange takes place between them, and ultimately the composition of the gas on both sides of the diaphragm is the same; but the rapidity with which different gases diffuse into each other under these circumstances varies considerably. The laws regulating this phenomenon have been investigated by Graham. Numerous experiments illustrate it, two of the most interesting of which are the following:—

A glass cylinder closed at one end is filled with carbonic acid gas, its open end tied over with a bladder, and the whole placed under a jar of hydrogen. Diffusion takes place between them through the porous diaphragm, and after the lapse of a certain time hydrogen has passed through the bladder into the cylindrical vessel in much greater quantity than the carbonic acid which has passed out, so that the bladder becomes very much distended outwards (fig. 115). If the cylinder be filled with hydrogen and

Fig. 115.

Fig. 116.

the bell-jar with carbonic acid, the reverse phenomenon will be produced—the bladder will be distended inwards (fig. 116).

A tube about 12 inches long, closed at one end by a plug of dry plaster of Paris, is filled with dry hydrogen, and its open end then immersed in a mercury bath. Endosmose of the hydrogen towards the air takes place so rapidly that a partial vacuum is produced, and mercury rises in the tube to a height of several inches (fig. 117). If several such tubes are filled with different gases, and allowed to diffuse into the air in a similar manner, in the same time, different quantities of the various gases will diffuse, and Graham found that the law regulating these diffusions is that *the force of diffusion is inversely as the square roots of the densities of gases.* Thus, if two vessels of equal capacity, containing oxygen and hydrogen, be separated by a porous plug, diffusion takes place; and after the lapse of some time, for every one part of oxygen which has passed into the hydrogen, four parts of hydrogen have passed into the oxygen. Now the density of hydrogen being 1, that of oxygen is 16, hence the force of diffusion is

Fig. 117.

inversely as the square roots of these numbers. It is four times as great in the one which has $\frac{1}{16}$ the density of the other.

Let the stem of an ordinary tobacco pipe be cemented, so that its ends project, in an outer glass tube, which can be connected with an air-pump and thus exhausted. On allowing then a slow current of air to enter one end of the pipe, its nitrogen diffuses more rapidly on its way through the porous pipe than the heavier oxygen, so that the gas which emerges at the other end, and which can be collected, is much richer in oxygen.

143. **Effusion and transpiration of gases.**—A gas can only flow from one space to another space occupied by the same gas when the pressure in the one is greater than in the other. *Effusion* is the term applied to the phenomenon of the passage of gases into vacuum, through a minute aperture not much more or less than 0·013 millimetre in diameter, in a thin plate of metal or of glass; for in a tube the friction of gases comes into play, and in a larger aperture the particles would strike against one another and form eddies and whirlpools. The velocity of the efflux is measured by the formula $v = \sqrt{2gh}$, in which h represents the pressure under which the gas flows, expressed in terms of the height of a column of the gas, which would exert the same pressure as that of the effluent gas. Thus for air under the ordinary pressure flowing into a vacuum, the pressure is equivalent to a column of mercury 76 centimetres high; and as mercury is approximately 10,500 times as dense as air, the equivalent column of air will be 76 centimetres $\times$ 10,500 = 7,980 metres. Hence the velocity of efflux of air into vacuum is $= \sqrt{2 \times 9{\cdot}8 \times 7{,}980} = 395{\cdot}5$ metres. This velocity into vacuum only holds, however, for the first moment, for the space contains a continually-increasing quantity of air, so that the velocity becomes continually smaller, and is null when the pressure on each side is the same. If the height of the column of air hh_1, corresponding to the external pressure, is known, the velocity may be calculated by the formula $v = \sqrt{2g(h - h_1)}$.

For gases lighter than air a greater height must be inserted in the formula, and for heavier gases a lower height; and this change must be inversely as the change of density. Hence *the velocities of efflux of various gases must be inversely as the square roots of their densities.* A simple inversion of this statement is that the *densities of two gases* are inversely as the squares of *their velocities of effusion.* On this Bunsen has based an interesting method of determining the densities of gases and vapours.

If gases issue through long, fine capillary tubes into a vacuum, the rate of efflux, or the *velocity of transpiration*, is independent of the rate of diffusion.

i. *For the same gas, the rate of transpiration increases, other things being equal, directly as the pressure*; that is, equal volumes of air of different densities require times inversely proportional to their densities.

ii. *With tubes of equal diameters, the volume transpired in equal times is inversely as the length of the tube.*

iii. *As the temperature rises the transpiration becomes slower.*

iv. *The rate of transpiration is independent of the material of the tube.*

144. **Absorption of gases.**—The surfaces of all solid bodies exert an attraction on the molecules of gases with which they are in contact, of such a nature that they become covered with a more or less thick layer of *con-*

densed gas. When a porous body such as a piece of charcoal, which consequently presents an immensely increased surface in proportion to its size, is placed in a vessel of ammonia gas over mercury (fig. 118), the great diminution of volume which ensues indicates that considerable quantities of gas are absorbed.


Fig. 118.

Now, although there is no absorption such as arises from chemical combinations between the solid and the gas (as with phosphorus and oxygen), still the quantity of gas absorbed is not entirely dependent on the physical conditions of the solid body; it is influenced in some measure by the chemical nature both of the solid and the gas. Boxwood charcoal has very great absorptive power. The following table gives the volumes of gas, under standard conditions of temperature and pressure, absorbed by one volume of boxwood charcoal and of meerschaum respectively:—

	Charcoal	Meerschaum.
Ammonia	90	15
Hydrochloric acid	85	—
Sulphurous acid	65	—
Sulphuretted hydrogen	55	11
Carbonic acid	35	5·3
Carbonic oxide	9·4	1·2
Oxygen	9·2	1·5
Nitrogen	7·5	1·6
Hydrogen	1·75	0·5

The absorption of gases is in general greatest in the case of those which are most easily liquefied.

Cocoanut charcoal is even more highly absorbent; it absorbs 171 of ammonia, 73 of carbonic acid, and 108 of cyanogen at the ordinary pressure; the amount of absorption increases with the pressure.

The absorptive power of pine charcoal is about half as much as that of boxwood. The charcoal made from corkwood, which is very porous, is not absorbent, neither is graphite. Platinum, in the finely divided form known as platinum sponge, is said to absorb 250 times its volume of oxygen gas. Many other porous substances, such as meerschaum, gypsum, silk, &c., are also highly absorbent.

If a coin be laid on a plate of glass or of metal, after some time, when the plate is breathed on, an image of the coin appears. If a figure is traced on a glass plate with the finger, nothing appears until the plate is breathed on, when the figure is at once seen. Indeed, the traces of an engraving which has long laid on a glass plate may be produced in this way.

These phenomena are known as *Moser's images*, for he first investigated them, although he explained them erroneously. The correct explanation was given by Waidele, who ascribed them to alterations in the layer of gas, vapour, and fine dust which is condensed on the surface of all solids. If

this layer is removed by wiping, on afterwards breathing against the surface more vapour is condensed on the marks in question, which then present a different appearance to the rest.

If a die or a stamp is laid on a freshly polished metal plate, and one therefore which has been deprived of its atmosphere, the layer of vapour from the coin will diffuse on to the metal plate, which thereby becomes altered; so that when this is breathed on an impression is seen.

Conversely, if a coin be polished and placed on an ordinary plate, it will partially remove the layer of gas from the parts in contact, so that on breathing on the plate the image is seen.

145. **Occlusion.**—Graham found that at a high temperature platinum and iron allow hydrogen to traverse them even more readily than does caoutchouc in the cold. Thus while a square metre of caoutchouc 0·014 millimetres in thickness allowed 129 cubic centimetres of hydrogen at 20° to traverse it in a minute, a platinum tube 1·1 millimetres in thickness and of the same surface allowed 489 cubic centimetres to traverse it at a bright red heat.

This is probably connected with the property which some metals, though destitute of physical pores, possess of absorbing gases either on their surface or in their mass, and to which Graham has applied the term *occlusion.* It is best observed by allowing the heated metal to cool in contact with the gas. The gas cannot then be extracted by the air-pump, but is disengaged on heating. In this way Graham found that platinum occluded four times its volume of hydrogen: iron wire 0·44 times its volume of hydrogen, and 4·15 volumes of carbonic oxide; silver reduced from the oxide, absorbed about seven volumes of oxygen, and nearly one volume of hydrogen when heated to dull redness in these gases. This property is most remarkable in palladium, which absorbs hydrogen, not only in cooling after being heated, but also in the cold. When, for instance, a palladium electrode is used in the decomposition of water, one volume of the metal can absorb 980 times its volume of the gas. This gas is again driven out on being heated, in which respect there is a resemblance to the solution of gases in liquids. By the occlusion of hydrogen the volume of palladium is increased by 0·09827 of its original amount, from which it follows that the hydrogen, which under ordinary circumstances has a density 0·000089546 that of water, has here a density nearly 9,868 times as great, or about 0·88 that of water. Hence the hydrogen must be in the liquid or even solid state; it probably forms thus an alloy with palladium, like a true metal—a view of this gas which is strongly supported by independent chemical considerations. The physical properties, in so far as they have been examined, support this view of its being an alloy.

BOOK IV.

ON GASES.

CHAPTER I.

PROPERTIES OF GASES. ATMOSPHERE. BAROMETERS.

146. **Physical properties of gases.**—Gases are bodies whose molecules are in a constant state of motion, in virtue of which they possess the most perfect mobility, and are continually tending to occupy a greater space. This property of gases is known by the names *expansibility*, *tension*, or *elastic force*, from which they are often called *elastic fluids*.

Gases and liquids have several properties in common, and some in which they seem to differ are in reality only different degrees of the same property. Thus, in both, the particles are capable of moving: in gases quite freely; in liquids not quite freely, owing to a certain degree of viscosity. Both are compressible, though in very different degrees. If a liquid and a gas both exist under the pressure of one atmosphere, and then the pressure be doubled, the water is compressed by about the $\frac{1}{20000}$ part, while the gas is compressed by one-half. In density there is a great difference; water, which is the type of liquids, is 770 times as heavy as air, the type of gaseous bodies, while under the pressure of one atmosphere. The property by which gases are distinguished from liquids is their tendency to indefinite expansion.

Matter assumes the solid, liquid, or gaseous form according to the relative strength of the cohesive and repulsive forces exerted between their molecules. In liquids these forces balance; in gases repulsion (287) preponderates.

By the aid of pressure and of low temperatures, the force of cohesion may be so far increased in many gases that they are readily converted into liquids, and we know now that with sufficient pressure and cold they may all be liquefied. On the other hand, heat, which increases the *vis viva* of the molecules, converts liquids, such as water, alcohol, and ether, into the aëriform state in which they obey all the laws of gases. This aëriform state of liquids is known by the name of *vapour*; while gases are bodies which, under ordinary temperature and pressure, remain in the aëriform state.

In describing the properties of gases we shall, for obvious reasons, have exclusive reference to atmospheric air as their type.

147. **Expansibility of gases.**—This property of gases, their tendency to assume continually a greater volume, is exhibited by means of the following

experiment :—A bladder, closed by a stopcock and about half-full of air, is placed under the receiver of the air-pump (fig. 119), and a vacuum is produced, on which the bladder immediately distends. This arises from the fact that the molecules of air flying about in all directions press against the sides of the bladder. Under ordinary conditions, this internal pressure is counterbalanced by the air in the receiver, which exerts an equal and contrary pressure. But when this pressure is removed by exhausting the receiver, the internal pressure becomes evident. When air is admitted into the receiver, the bladder resumes its original form.

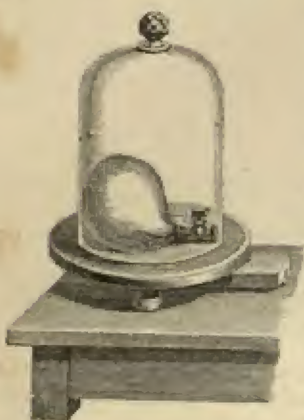

Fig. 119.

148. **Compressibility of gases.**—The compressibility of gases is readily shown by the *pneumatic syringe* (fig. 120). This consists of a stout glass tube closed at one end and provided with a tight-fitting solid piston. When the rod of the piston is pressed, it moves down in the tube, and the air becomes compressed into a smaller volume ; but as soon as the force is removed the air regains its original volume, and the piston rises to its former position.

Fig. 120.

149. **Weight of gases.**—From their extreme fluidity and expansibility, gases seem to be uninfluenced by the force of gravity : they nevertheless possess weight like solids and liquids. To show this, a glass globe of 3 or 4 quarts capacity is taken (fig. 121), the neck of which is provided with a stopcock, which hermetically closes it and by which it can be screwed to the plate of the air-pump. The globe is then exhausted, and its weight determined by means of a delicate balance. Air is now allowed to enter, and the globe again weighed. The weight in the second case will be found to be greater than before, and, if the capacity of the vessel is known, the increase will obviously be the weight of that volume of air.

By a modification of this method, and with the adoption of certain precautions, the weight of air and of other gases has been determined. Perhaps the most accurate are those of Regnault, who found that a litre of dry air at 0° C., and under a pressure of 760 millimetres, weighs 1·293187 grammes. Since a litre of water (or 1,000 cubic centimetres) at 0° weighs 0·999877

grammes, the density of air is 0·00129334 that of water under the same circumstances; that is, water is 773 times as heavy as air. Expressed in English measures, 100 cubic inches of dry air under the ordinary atmospheric pressure of 30 in. and at the temperature of 16° C. weigh 31 grains; the same volume of carbonic acid gas under the same circumstances weighs 47·25 grains; 100 cubic inches of hydrogen, the lightest of all gases, weigh 2·14 grains; and 100 cubic inches of hydriodic acid gas weigh 146 grains.

Fig. 121.

150. **Pressures exerted by gases.**—Gases exert on their own molecules and on the sides of vessels which contain them, pressures which may be regarded from two points of view. First, we may neglect the weight of the gas; secondly, we may take account of its weight. If we neglect the weight of any gaseous mass at rest, and only consider its expansive force, it will be seen that the pressures due to this force act with the same intensity on all points, both of the mass itself and of the vessel in which it is contained. For it is a necessary consequence of the elasticity and fluidity of gases, that the repulsive force between the molecules is the same at all points, and acts equally in all directions. This principle of the equality of the pressure of gases in all directions may be shown experimentally by means of an apparatus resembling that by which the same principle is demonstrated for liquids (fig. 66).

If we consider the weight of any gas we shall see that it gives rise to pressures which obey the same laws as those produced by the weight of liquids. Let us imagine a cylinder, with its axis vertical, several miles high, closed at both ends and full of air. Let us consider any small portion of the air enclosed between two horizontal planes. This portion must sustain the weight of all the air above it, and transmit that weight to the air beneath it, and likewise to the curved surface of the cylinder which contains it, and at each point in a direction at right angles to the surface. Thus the pressure increases from the top of the column to the base; at any given layer, it acts equally on equal surfaces, and at right angles to them, whether they are horizontal, vertical, or inclined. The pressure acts on the sides of the vessel, and on any small surface it is equal to the weight of a column of gas, whose base is this surface, and whose height its distance from the summit of the column. The pressure is also independent of the shape and dimensions of the supposed cylinder, provided the height remains the same.

For a small quantity of gas the pressures due to its weight are quite insignificant, and may be neglected; but for large quantities, like the atmosphere, the pressures are considerable, and must be allowed for.

151. **The atmosphere. Its composition.**—The atmosphere is the layer of air which surrounds our globe in every part. It partakes of the rotatory motion of the globe, and would remain fixed relatively to terrestrial objects but for local circumstances, which produce winds, and are constantly disturbing its equilibrium.

It is essentially a mixture of oxygen and nitrogen gases; its average composition by volume being as follows:—

Nitrogen	78·49
Oxygen	20·63
Aqueous vapour	0·84
Carbonic acid	0·04
	100·00

The carbonic acid arises from the respiration of animals, from the processes of combustion, and from the decomposition of organic substances. Boussingault has estimated that in Paris the following quantities of carbonic acid are produced every 24 hours:—

By the population and by animals	11,895,000 cubic feet
By processes of combustion	92,101,000 ,,
	103,996,000

Notwithstanding this enormous continual production of carbonic acid the composition of the atmosphere does not vary; for plants in the process of vegetation decompose the carbonic acid, assimilating the carbon, and restoring to the atmosphere the oxygen, which is being continually consumed in the processes of respiration and combustion.

152. **Atmospheric pressure.**—If we neglect the perturbations to which the atmosphere is subject, as being inconsiderable, we may consider it as a fluid sea of a certain depth, surrounding the earth on all sides, and exercising the same pressure as if it were a liquid of very small density. Consequently, the pressure on the unit of area is constant at a given level, being equal to the weight of the column of atmosphere above that level whose horizontal section is the unit of area. It will act at right angles to the surface, whatever be its position. It will diminish as we ascend, and increase as we descend from that level. Consequently, at the same height, the atmospheric pressures on unequal plane surfaces will be proportional to the areas of those surfaces, provided they be small in proportion to the height of the atmosphere.

In virtue of the expansive force of the air, it might be supposed that the molecules would expand indefinitely into the planetary spaces. But, in proportion as the air expands, its expansive force decreases, and is further weakened by the low temperature of the upper regions of the atmosphere, so that, at a certain height, an equilibrium is established between the expansive force which separates the molecules, and the action of gravity which draws them towards the centre of the earth. It is therefore concluded that the atmosphere is limited.

From the weight of the atmosphere, and its increase in density, and from the observation of certain phenomena of twilight, its height has been estimated at from 30 to 40 miles. Above that height the air is extremely rarefied, and at a height of 60 miles it is assumed that there is a perfect vacuum. On the other hand, meteorites have been seen at a height of 200 miles, and as their luminosity is undoubtedly due to the action of air, there must be air at such a height. This higher estimate is supported by observations made at Rio Janeiro on the twilight arc, by M. Liais, who estimates the height of the atmosphere at between 198 and 212 miles. The question as to the exact height of the atmosphere must therefore be considered as still awaiting settlement.

As it has been previously stated that 100 cubic inches of air weigh 31 grains, it will readily be conceived that the whole atmosphere exercises a considerable pressure on the surface of the earth. The existence of this pressure is shown by the following experiments.

153. **Crushing force of the atmosphere.**—On one end of a stout glass cylinder, about 5 inches high, and open at both ends, a piece of bladder is tied quite air-tight. The other end, the edge of which is ground and well greased, is pressed on the plate of the air-pump (fig. 122). As soon as the air in the vessel is rarefied, by working the air-pump, the bladder is depressed by the weight of the atmosphere above it, and finally bursts with a loud report caused by the sudden entrance of the air.

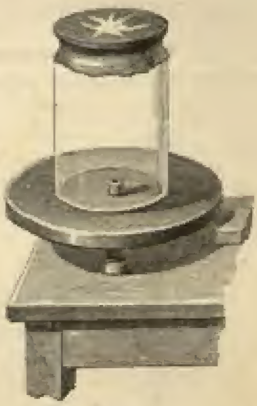

Fig. 122.

154. **Magdeburg hemispheres.**—The preceding experiment only serves to illustrate the downward pressure of the atmosphere. By means of the *Magdeburg hemispheres* (figs. 123 and 124), the invention of which is due to Otto von Guericke, burgomaster of Magdeburg, it can be shown that the pressure acts in all directions. This apparatus consists of two hollow brass hemispheres of 4 to $4\frac{1}{2}$ inches diameter, the edges of which are made to fit tightly, and are well greased. One of the hemispheres is provided with a stopcock, by which it can be screwed on the air-pump, and on the other there

Fig. 123.

Fig. 124.

is a handle. As long as the hemispheres contain air they can be separated without any difficulty, for the external pressure of the atmosphere is counter-

balanced by the elastic force of the air in the interior. But when the air in the interior is pumped out by means of the air-pump, the hemispheres cannot be separated without a powerful effort; and as this is the case in whatever position they are held, it follows that the atmospheric pressure is transmitted in all directions.

DETERMINATION OF THE ATMOSPHERIC PRESSURE. BAROMETERS.

155. **Torricelli's experiment.**—The above experiments demonstrate the existence of the atmospheric pressure, but they give no precise indications as to its amount. The following experiment, which was first made, in 1643, by Torricelli, a pupil of Galileo, gives an exact measure of the weight of the atmosphere.

Fig. 125.

A glass tube is taken, about a yard long and a quarter of an inch internal diameter (fig. 125). It is sealed at one end, and is quite filled with mercury. The aperture C being closed by the thumb, the tube is inverted, the open end placed in a small mercury trough, and the thumb removed. The tube being in a vertical position, the column of mercury sinks, and, after oscillating some time, it finally comes to rest at a height A, which at the level of the sea is about 30 inches above the mercury in the trough. The mercury is raised in the tube by the pressure of the atmosphere on the mercury in the trough. There is no contrary pressure on the mercury in the tube, because it is closed. But if the end of the tube be opened, the atmosphere will press equally inside and outside the tube, and the mercury will sink to the level of that in the trough. It has been shown in hydrostatics (108) that the heights of two columns of liquid in communication with each other are inversely as their densities, and hence it follows that the pressure of the atmosphere is equal to that of a column of mercury, the height of which is 30 inches. If, however, the weight of the atmosphere diminishes, the height of the column which it can sustain must also diminish.

156. **Pascal's experiments.**—Pascal, who wished to ascertain whether the force which sustained the mercury in the tube was really the pressure of the atmosphere, made the following experiments. i. If it were the case, the column of mercury ought to descend in proportion as we ascend in the atmosphere. He accordingly requested one of his relations to repeat Torricelli's experiment on the summit of the Puy de Dôme in Auvergne.

This was done, and it was found that the mercurial column was about 3 inches lower, thus proving that it is really the weight of the atmosphere which supports the mercury, since, when this weight diminishes, the height of the column also diminishes. ii. Pascal repeated Torricelli's experiment at Rouen, in 1646, with other liquids. He took a tube closed at one end, nearly 50 feet long, and, having filled it with water, placed it vertically in a vessel of water, and found that the water stood in the tube at a height of 34 feet; that is, 13·6 times as high as mercury. But since mercury is 13·6 times as heavy as water, the weight of the column of water was exactly equal to that of the column of mercury in Torricelli's experiment, and it was consequently the same force, the pressure of the atmosphere, which successively supported the two liquids. Pascal's other experiments with oil and with wine gave similar results.

157. **Amount of the atmospheric pressure.**—Let us assume that the tube in the above experiment is a cylinder, the section of which is equal to a square inch, then, since the height of the mercurial column in round numbers is 30 inches, the column will contain 30 cubic inches, and as a cubic inch ot mercury weighs 3433·5 grains = 0·49 of a pound, the pressure of such a column on a square inch of surface is equal to 14·7 pounds. In round numbers the pressure of the atmosphere is taken at 15 pounds on the square inch. A surface of a foot square contains 144 square inches, and therefore the pressure upon it is equal to 2,160 pounds, or nearly a ton. Expressed in the metrical system, the standard atmospheric pressure at 0° and the sea level is 760 millimetres, which is equal to 29·9217 inches; and a calculation similar to the above shows that the pressure on a square centimetre is = 1·03296 kilogramme.

A gas or liquid which acts in such a manner that a square inch of surface is exposed to a pressure of 15 pounds, is called a pressure of *one atmosphere.* If, for instance, the elastic force of the steam of a boiler is so great that each square inch of the internal surface is exposed to a pressure of 90 pounds ($=6 \times 15$), we say it is under a pressure of six atmospheres.

The surface of the body of a man of middle size is about 16 square feet; the pressure, therefore, which a man supports on the surface of his body is 35,560 pounds, or nearly 16 tons. Such an enormous pressure might seem impossible to be borne; but it must be remembered that, in all directions, there are equal and contrary pressures which counterbalance one another. It might also be supposed that the effect of this force, acting in all directions, would be to press the body together and crush it. But the solid parts of the skeleton could resist a far greater pressure; and as to the air and liquids contained in the organs and vessels, the air has the same density as the external air, and cannot be further compressed by the atmospheric pressure; and from what has been said about liquids (98), it is clear that they are virtually incompressible. When the external pressure is removed from any part of the body, either by means of a cupping vessel or by the air-pump, the pressure from within is seen by the distension of the surface.

158. **Different kinds of barometers.**—The instruments used for measuring the atmospheric pressure are called *barometers.* In ordinary barometers, the pressure is measured by the height of a column of mercury, as in Torricelli's experiment: the barometers which we are about to describe

are of this kind. But there are barometers without any liquid, one of which, the aneroid (181), is remarkable for its simplicity and portability.

159. **Cistern barometer.**—The *cistern barometer* consists of a straight glass tube closed at one end, about 33 inches long, filled with mercury, and dipping into a cistern containing the same metal. In order to render the barometer more portable, and the variations of the level in the cistern less perceptible when the mercury rises or falls in the tube, several different

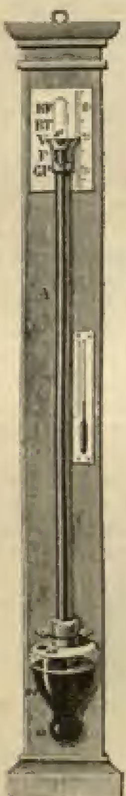

Fig. 126.

Fig. 127.

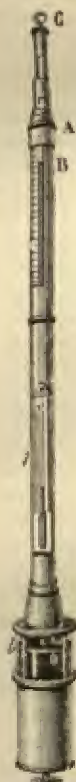

Fig. 128.

forms have been constructed. Fig. 126 represents one form of the cistern barometer. The apparatus is fixed to a mahogany stand, on the upper part of which there is a scale graduated in millimetres or inches from the level of the mercury in the cistern; a movable index, *i*, shows on the scale the level of the mercury. A thermometer on one side of the tube indicates the temperature.

There is one fault to which this barometer is liable, in common with all others of the same kind. The zero of the scale does not always correspond

to the level of the mercury in the cistern. For, as the atmospheric pressure is not always the same, the height of the mercurial column varies; sometimes mercury is forced from the cistern into the tube, and sometimes from the tube into the cistern, so that, in the majority of cases, the graduation of the barometer does not indicate the true height. If the diameter of the cistern is large, relatively to that of the tube, the error from this source is lessened. The *height* of the barometer is the distance between the levels of the mercury in the tube and in the cistern. Hence the barometer should always be perfectly vertical, for, if not, the tube being inclined, the column of mercury is elongated (fig. 127), and the number read off on the scale is too great. As the pressure which the mercury exerts by its weight at the base of the tube is independent of the form of the tube and of its diameter (102), provided it is not capillary, the height of the barometer is independent of the diameter of the tube and of its shape, but is inversely as the density of the liquid. With mercury the mean height at the level of the sea is 29·92, or in round numbers 30, inches; in a water barometer it would be about 34 feet, or 10·33 metres.

The 'Philosophical Magazine,' vol. xxx. Fourth Series, page 349, contains a detailed account of a method of constructing a water barometer.

160. **Fortin's barometer.**—*Fortin's barometer* differs from that just described, in the shape of the cistern. The base of the cistern is made of leather, and can be raised or lowered by means of a screw: this has the advantage, that a constant level can be obtained, and also that the instrument is made more portable. For, in travelling, it is only necessary to raise the leather until the mercury, which rises with it, quite fills the cistern; the barometer may then be inclined, and even inverted, without any fear that a bubble of air may enter, or that the shock of the mercury may crack the tube.

Fig. 128 represents the arrangement of the barometer, the tube of which is placed in a brass case. At the top of this case there are two longitudinal apertures, on opposite sides, so that the level of the mercury, B, is seen. The scale on the case is graduated in millimetres. An index A, moved by the hand, gives, by means of a vernier, the height of the mercury to $\frac{1}{10}$th of a millimetre. At the bottom of the case there is a cistern *b*, containing mercury, O.

Fig. 129 shows the details of the cistern on a larger scale. It consists of a glass cylinder *b*, through which the mercury can be seen; this is closed at the top by a box-wood disc fitted on the under surface of the brass cover M. Through this passes the barometer tube E, which is drawn out at the end, and dips in the mercury; the cistern and the tube are connected by a piece of buckskin *ce*, which is firmly tied at *c* to a contraction in the tube, and at *e* to a brass tubulure in the cover of the cistern. This mode of closing prevents the mercury from escaping when the barometer is inverted, while the pores of the leather transmit the atmospheric pressure. The bottom of the cylinder *b* is cemented on a box-wood cylinder *zz*, on a contraction in which, *ii*, is firmly tied the buckskin *mn*, which forms the base of the cistern. On this skin is fastened a wooden button *x*, which rests against the end of a screw C. According as this is turned in one direction or the other, the skin *mn* is raised or lowered, and with it the mercury. In using this baro-

meter the mercury is first made exactly level with the point *a*, which is effected by turning the screw C either in one direction or the other. The graduation of the scale is counted from this point *a*, and thus the distance of the top B of the column of mercury from *a* gives the height of the barometer. The bottom of the cistern is surrounded by a brass case, which is fastened to the cover M by screws, *k*, *k*, *k*. We have already seen (159) the importance of having the barometer quite vertical, which is effected by the following plan, known as *Cardan's suspension.*

The metal case containing the barometer is filled in a copper sheath X by two screws *a* and *b* (fig. 130). This is provided with two axles (only one

Fig. 129.

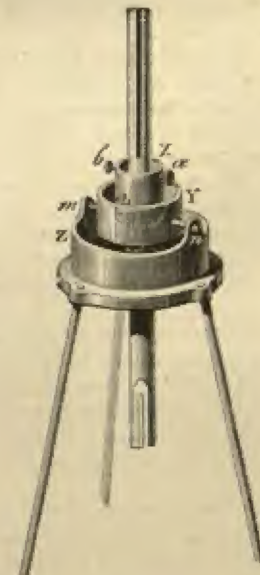

Fig. 130.

of which, *o*, is seen in the figure), which turn freely in two holes in a ring Y. In a direction at right angles to that of the axles, *oo*, the ring has also two similar axles, *m* and *n*, resting on a support Z. By means of this double suspension, the barometer can oscillate freely about the axes, *mn* and *oo*, in two directions at right angles to each other. But as care is taken that the point at which these axes cross corresponds to the tube itself, the centre of gravity of the system, which must always be lower than the axis of suspension, is below the point of intersection, and the barometer is then perfectly vertical.

161. **Gay-Lussac's syphon barometer.**—The syphon barometer is a bent glass tube, one of the branches of which is much longer than the other. The longer branch, which is closed at the top, is filled with mercury as in the cistern barometer, while the shorter branch, which is open, serves as a cistern. The difference between the two levels is the height of the barometer.

Fig. 131 represents the syphon barometer as modified by Gay-Lussac. In order to render it more available for travelling by preventing the entrance of air, he joined the two branches by a capillary tube (fig. 132); when the

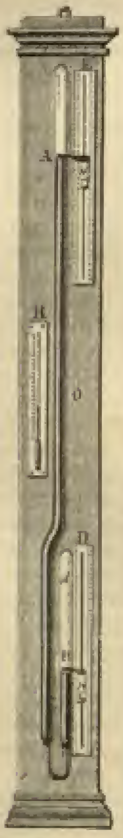

Fig. 131.

Fig. 132.

Fig. 133.

Fig. 134.

instrument is inverted (fig. 133) the tube always remains full in virtue of its capillarity, and air cannot penetrate into the longer branch. A sudden shock, however, might separate the mercury and admit some air. To avoid this, M. Bunten has introduced an ingenious modification into the apparatus. The longer branch is drawn out to a fine point, and is joined to a tube B of

the form represented in fig. 134. By this arrangement, if air passes through the capillary tube it cannot penetrate the drawn-out extremity of the longer branch, but lodges in the upper part of the enlargement B. In this position it does not affect the observations, since the vacuum is always at the upper part of the tube; it is, moreover, easily removed.

In Gay-Lussac's barometer the shorter branch is closed, but there is a capillary aperture in the side *i*, through which the atmospheric pressure is transmitted.

The barometric height is determined by means of two scales, which have a common zero at O, towards the middle of the longer branch, and are graduated in contrary directions, the one from O to E, and the other from O to B, either on the tube itself, or on brass rules fixed parallel to the tube. Two sliding verniers, *m* and *n*, indicate tenths of a millimetre. The total height of the barometer, AB, is the sum of the distances from O to A and from O to B.

162. **Precautions in reference to barometers.**—In constructing barometers, mercury is chosen in preference to any other liquid. For being the densest of all liquids, it stands at the least height. When the mercurial barometer stands at 30 inches, the water barometer would stand at about 34 feet (159). It also deserves preference because it does not moisten the glass. It is necessary that the mercury be pure and free from oxide, otherwise it adheres to the glass and tarnishes it. Moreover, if it is impure its density is changed, and the height of the barometer is too great or too small. Mercury is purified, before being used for barometers, by treatment with dilute nitric acid, and by distillation.

The space at the top of the tube (figs. 126 and 131), which is called the *Torricellian vacuum*, must be quite free from air and from aqueous vapour, for otherwise either would depress the mercurial column by its elastic force. To obtain this result, a small quantity of pure mercury is placed in the tube and boiled for some time. It is then allowed to cool, and a further quantity, previously warmed, added, which is boiled, and so on, until the tube is quite full; in this manner the moisture and the air which adhere to the sides of the tube (144) pass off with the mercurial vapour. A barometer tube should not be too narrow, for otherwise the mercury is moved with difficulty; and before reading off, the barometer should be tapped so as to get rid of the adhesion to the glass.

A barometer is free from air and moisture if, when it is inclined, the mercury strikes with a sharp metallic sound against the top of the tube. If there is air or moisture in it, the sound is deadened.

Fig. 135.

163. **Correction for capillarity.**—In cistern barometers there is always a certain depression of the mercurial column due to capillarity, unless the internal diameter of the tube exceeds 0·8 inch. To make the correction due to this depression, it is not enough to know the diameter of the tube; we must also know the height of the meniscus *od* (fig. 135), which varies according as the meniscus has been formed during an ascending or descending motion of the mercury in the tube. Consequently the height of the meniscus must be determined by

bringing the pointer to the level *ab*, and then to the level *d*, when the difference of the readings will give the height *od* required. These two terms—namely, the internal diameter of the tube and the height of the meniscus—being known, the resulting correction can be taken out of the following table :

Internal Diameter in inches	Height of Sagitta of Meniscus in inches						
	0·010	0·015	0·020	0·025	0·030	0·035	0·040
0·157	0·0293	0·0431	0·0555	0·0677	0·0780	0·0870	0·0948
0·236	0·0119	0·0176	0·0231	0·0294	0·0342	0·0398	0·0432
0·315	0·0060	0·0088	0·0118	0·0144	0·0175	0·0196	0·0221
0·394	0·0039	0·0048	0·0063	0·0078	0·0095	0·0110	0·0125
0·472	0·0020	0·0029	0·0036	0·0045	0·0053	0·0063	0·0073
0·550	0·0010	0·0017	0·0024	0·0029	0·0034	0·0039	0·0044

In Gay-Lussac's barometer the two tubes are made of the same diameter, so that the error caused by the depression in the one tube very nearly corrects that caused by the depression in the other. As, however, the meniscus in the one tube is formed by a column of mercury with an ascending motion, while that in the other is formed by a column with a descending motion, their heights will not be the same, and the reciprocal correction will not be quite exact.

164. **Correction for temperature.**—In all observations with barometers, whatever be their construction, a correction must be made for temperature. Mercury contracts and expands with different temperatures; hence its density changes, and consequently the barometric height, for this height is inversely as the density of the mercury, so that for different atmospheric pressures the mercurial column might have the same height. Accordingly, in each observation, the height observed must be reduced to a determinate temperature. The choice of this is quite arbitrary, but that of melting ice is in practice always adopted. It will be seen, in the Book on Heat, how this correction is made.

165. **Variations in the height of the barometer.**—When the barometer is observed for several days, its height is found to vary in the same place, not only from one day to another, but also during the same day.

The extent of these variations—that is, the difference between the greatest and the least height—is different in different places. It increases from the equator towards the poles. Except under extraordinary circumstances, the greatest variations do not exceed six millimetres under the equator, 30 under the tropic of Cancer, 40 in France, and 60 at 25 degrees from the pole. The greatest variations are observed in winter.

The *mean daily height* is the height obtained by dividing the sum of 24 successive hourly observations by 24. In our latitudes the barometric height at noon corresponds to the mean daily height.

The *mean monthly height* is obtained by adding together the mean daily heights for a month, and dividing by 30. The *mean yearly height* is similarly obtained.

Under the equator, the mean annual height at the level of the sea is $0^m.758$, or 29·84 inches. It increases from the equator, and between the latitudes 30° and 40° it attains a maximum of $0^m.763$, or 30·04 inches. In lower latitudes it decreases, and in Paris it does not exceed $0^m.7568$.

The general mean at the level of the sea is $0^m.761$, or 29·96 inches.

The mean monthly height is greater in winter than in summer, in consequence of the cooler atmosphere.

Two kinds of variations are observed in the barometer:—1st, the *accidental variations*, which present no regularity; they depend on the seasons, the direction of the winds, and the geographical position, and are common in our climates; 2nd, the *daily variations*, which are produced periodically at certain hours of the day.

At the equator, and between the tropics, no accidental variations are observed; but the daily variations take place with such regularity that a barometer may serve to a certain extent as a clock. The barometer sinks from midday till towards four o'clock; it then rises, and reaches its maximum at about ten o'clock in the evening. It then again sinks, and reaches a second minimum towards four o'clock in the morning, and a second maximum at ten o'clock.

In the temperate zones there are also daily variations, but they are detected with difficulty, since they occur in conjunction with accidental variations.

The hours of the maxima and minima appear to be the same in all climates, whatever be the latitude; they merely vary a little with the seasons.

166. **Causes of barometric variations.**—It is observed that the course of the barometer is generally in the opposite direction to that of the thermometer; that is, that when the temperature rises the barometer falls, and *vice versâ*; which indicates that the barometric variations at any given place are produced by the expansion or contraction of the air, and therefore by its change in density. If the temperature were the same throughout the whole extent of the atmosphere, no currents would be produced, and, at the same height, atmospheric pressure would be everywhere the same. But when any portion of the atmosphere becomes warmer than the neighbouring parts, its specific gravity is diminished, and it rises and passes away through the upper regions of the atmosphere, whence it follows that the pressure is diminished, and the barometer falls. If any portion of the atmosphere retains its temperature, while the neighbouring parts become cooler, the same effect is produced; for in this case, too, the density of the first-mentioned portion is less than that of the others. Hence, also, it usually happens that an extraordinary fall of the barometer at one place is counterbalanced by an extraordinary rise at another place. The daily variations appear to result from the expansions and contractions which are periodically produced in the atmosphere by the heat of the sun during the rotation of the earth.

167. **Relation of barometric variations to the state of the weather.**—It has been observed that, in our climate, the barometer in fine weather is generally above 30 inches, and is below this point when there is rain, snow, wind, or storm, and also, that for any given number of days at which the barometer stands at 30 inches, there are as many fine as rainy days. From this coincidence between the height of the barometer and the state of the

weather, the following indications have been marked on the barometer, counting by thirds of an inch above and below 30 inches :—

Height	State of the weather
31 inches	Very dry.
$30\frac{2}{3}$ "	Settled weather.
$30\frac{1}{3}$ "	Fine weather.
30 "	Variable.
$29\frac{2}{3}$ "	Rain or wind.
$29\frac{1}{3}$ "	Much rain.
29 "	Tempest.

In using the barometer as an indicator of the state of the weather, we must not forget that it really only serves to measure the weight of the atmosphere, and that it only rises or falls as the weight increases or diminishes; and although a change of weather frequently coincides with a change in the pressure, they are not necessarily connected. This coincidence arises from meteorological conditions peculiar to our climate, and does not always occur. That a fall in the barometer usually precedes rain in our latitudes, is caused by the position of Europe. The south-west winds, which are hot and consequently light, make the barometer sink; but at the same time, as they become charged with aqueous vapour in crossing the ocean, they bring us rain. The winds of the north and north-east, on the contrary, being colder and denser, make the barometer rise; and as they only reach us after having passed over vast continents, they are generally dry.

When the barometer rises or sinks slowly, that is, for two or three days, towards fine weather or towards rain, it has been found from a great number of observations that the indications are then extremely probable. Sudden variations in either direction indicate bad weather or wind.

168. **Wheel barometer.**—The *wheel barometer*, which was invented by Hooke, is a syphon barometer, and is especially intended to indicate good and bad weather (fig. 136). In the shorter leg of the syphon there is a float which rises and falls with the mercury (fig. 137). A string attached to this float passes round a pulley, O, and at the other end there is a weight, P, somewhat lighter than the float. A needle fixed to the pulley moves round a graduated circle, on which is marked *variable*, *rain*, *fine weather*, &c. When the pressure varies the float sinks or rises, and moves the needle round to the corresponding points on the scale.

The barometers ordinarily met with in houses, and which are called *weather glasses*, are of this kind. They are, however, of little use, for two reasons. The first is, that they are neither very delicate nor very accurate in their indications. The second, which applies equally to all barometers, is that those commonly in use in this country are made in London, and the indications, if they are of any value, are only so for a place of the same level and of the same climatic conditions as London. Thus a barometer standing at a certain height in London would indicate a certain state of weather, but if removed to Shooter's Hill it would stand half an inch lower, and would indicate a different state of weather. As the pressure differs with the level and with geographical conditions, it is necessary to take these into account if exact data are wanted.

169. **Fixed barometer.**—For accurate observations Regnault uses a barometer the height of which he measures by means of a cathetometer (89). The cistern (fig. 138) is of cast iron; against the frame on which it is supported a screw is fitted, which is pointed at both ends, and the length of which has been determined, once for all, by the cathetometer. To measure the barometric height, the screw is turned until its point grazes the surface


Fig. 136


Fig. 137.


Fig. 138.

of the mercury in the bath, which is the case when the point and its image are in contact. The distance then from the top of the point to the level of the mercury in the tube *b* is measured by the cathetometer, and this, together with the length of the screw, gives the barometric height with great accuracy. This barometer has moreover the advantage that, as a tube an inch in diameter may be used, the influence of capillarity becomes inappreciable. Its construction, moreover, is very simple, and the position of the scale leads to no kind of error, since this is transferred to the cathetometer. Unfortunately the latter instrument requires great accuracy in its construction, and is very expensive.

170. **Glycerine barometer.**—Jordan has recently constructed a barometer in which the liquid used is pure glycerine. This has the specific gravity 1·26, and therefore the length of the column of liquid is rather more than ten times that of mercury; hence small alterations in the atmospheric pressure produce considerable oscillations in the height of the liquid. The tube consists of ordinary composition gas tubing about $\frac{4}{8}$ of an inch in diameter and 28 feet or so in length; the lower end is open and dips in the cistern, which may be placed in a cellar; the top is sealed to a closed glass tube an inch in diameter, in which the fluctuations of the column are observed. This may be arranged in an upper storey, and the tubing, being easily bent, lends itself to any adjustment which the locality requires.

The vapour of glycerine has very low tension at ordinary temperatures, and is therefore not so exposed to such back pressures, varying with the temperature, as is water. On the other hand, it readily attracts moisture from the air, whereby the density and therewith the height of the liquid column vary. This is prevented by covering the liquid in the cistern with a layer of paraffine oil.

171. **Huyghens' barometer.**—The desire to amplify the small variations which take place in the barometer has led to a number of contrivances, one of the best known of which was invented by Huyghens (fig. 139.)

Fig. 139.

The barometer tube *a* is wider at the closed end *b*, and also at *c*, where a liquid of smaller specific gravity than mercury, such as coloured water, is poured on the mercury; it fills the rest of the tube *c* and a portion of *d*.

Suppose *b* and *c* to have the same diameter, which is *n* times that of *d*. When the column of mercury in *b* sinks through *x* millimetres, the level of the mercury in *c* rises just as much, while the coloured liquid rises *nx* millimetres, and therefore its level is $(n-1)\ x$ millimetres higher. A column of this liquid $(n-1)\ x$ in height, has the same pressure as a column of mercury $\frac{(n-1)\ x}{s}$ in height where *s* is the number expressing the ratio of the specific gravities of mercury and the liquid.

When therefore the mercury in *b* sinks *x* millimetres,

$$y = 2x + \frac{n-1}{s}x$$

is the height of the column of mercury which corresponds to the decrease of atmospheric pressure. From this we have

$$x = \frac{sy}{2s+n-1}.$$

Thus, if the section of the tubes *b* and *c* is 20 times that of *d*, and if the coloured liquid be water, we have

$$\frac{13·6y}{27·2+20-1} = \frac{13·6y}{46·2} = 0·294y.$$

When, therefore, an ordinary barometer sinks through *y* millimetres, the

mercury in b sinks $0{\cdot}294y$ millimetres, while the coloured liquid rises $20 \times 0{\cdot}294y = 5{\cdot}88y$. Whenever, that is, an ordinary barometer sinks or rises 1 millimetre, the coloured liquid rises or sinks $5{\cdot}98$ millimetres, or nearly six times as much.

Such barometers are useful in cases where the variations in the height of the barometer, rather than its actual height, are to be observed. The scale should be placed behind the tube d and two points fixed, near the top and bottom, by comparison with standard barometers; the interval between the two is then suitably divided.

172. **Determination of heights by the barometer.**—Since the atmospheric pressure decreases as we ascend, it is obvious that the barometer will keep on falling as it is taken to a greater and greater height. On this depends a method of determining the difference between the heights of two stations, such as the base and summit of a mountain. The method may be explained as follows.

It will be seen in the next chapter that, according to Boyle's law, if the temperature of an enclosed portion of air continues constant, its volume will vary inversely as the pressure; that is to say, if we double the pressure we shall halve the volume. But if we halve the volume we manifestly double the quantity of air in each cubic inch—that is to say, we double the density of the air; and so on in any proportion. Consequently the law is equivalent to this:—*That for a constant temperature the density of air is proportional to the pressure which it sustains.*

Fig. 140.

Now suppose A and B (fig. 140) to represent two stations, and that it is required to determine the vertical height of B above A, it being borne in mind that A and B are not necessarily in the same vertical line. Take P, any point in AB, and Q, a point at a small distance above P. Suppose the pressure on a square inch of the atmosphere at P to be denoted by p, and at Q let it be diminished by a quantity denoted by dp. It is clear that this diminution equals the weight of the column of air between P and Q, whose section is one square inch. But, since the density of the air is directly proportional to p, the weight of a cubic inch of air will equal kgp, where k denotes a certain quantity to be determined presently, and g the accelerating force of gravity (80). Hence, if we denote PQ in inches by dx, the pressure will be diminished by $kpg \,.\, dx$, and we may represent this algebraically by the equation

$$kpg \,.\, dx = dp.$$

By a certain algebraical process this leads to the conclusion that

$$kgX = \log \frac{P}{P_1}$$

where X denotes the height of AB, and P and P_1 the atmospheric pressures at A and B respectively, the logarithms being what are called 'Napierian logarithms.' Now, if H and H_1 are the heights of the barometer at A and B respectively, the temperature of the mercury being the same at both stations, their ratio equals that of P to P_1, and therefore

$$X = \frac{1}{kg} \,.\, \log \frac{H}{H_1}$$

It remains to determine k and g.

(1) Since the force of gravity is different for places in different latitudes, g will depend upon the latitude (83). It is found that if g is the accelerating force of gravity in latitude ϕ, and f that force in latitude 45°, then

$$g = \frac{f}{1 + 0{\cdot}00256 \cos 2\phi}$$

where f has a definite numerical value.

(2) From what has been stated above it will be seen, that if ρ is the density of air at a temperature of $t°$ C., under Q, the pressure exerted by 29·92 inches of mercury, we shall have

$$k\mathrm{Q} = \rho.$$

But it will be afterwards shown that if ρ_0 is the density of air under the same pressure Q at 0° C., we shall have

$$\rho = \frac{\rho_0}{1 + at},$$

where a represents the coefficient of expansion of gases. Therefore

$$k\mathrm{Q} = \frac{\rho_0}{1 + at}$$

Now if σ is the density of mercury, and if the latitude is 45°, we shall have

$$\mathrm{Q} = 29{\cdot}92 \,.\, \sigma f;$$

and therefore

$$kf = \frac{\rho_0}{\sigma} \,.\, \frac{1}{29{\cdot}92\,(1 + at)}.$$

But $\rho_0 \div \sigma$ is the ratio which the density of dry air at a temperature 0° C., in latitude 45°, under a pressure of 29·92 inches of mercury, bears to the density of mercury at 0° C., and therefore $\rho_0 \div \sigma$ is a determinate number.

Substituting, we have

$$\mathrm{P} = 29{\cdot}92 \text{ in.} \,.\, \frac{\sigma}{\rho_0}(1 + 0{\cdot}00256 \cos 2\phi) \,.\, (1 + at) \log \frac{\mathrm{H}}{\mathrm{H}_1}.$$

The value of a is 0·003665, which is nearly equal to $\frac{11}{3000}$. If we substitute the proper values for $\sigma \div \rho_0$, and change the logarithms into common logarithms, and instead of t use the mean of T and T_1, the temperatures at the upper and lower stations, it will be found that

$$\mathrm{X} \text{ (in feet)} = 60346\,(1 + 0{\cdot}00256 \cos 2\phi) \left(1 + \frac{2(\mathrm{T} + \mathrm{T}_1)}{1000}\right) \log \frac{\mathrm{H}}{\mathrm{H}_1}$$

which is La Place's barometric formula. In using it, we must remember that T and T_1 are temperatures on the Centigrade thermometer, and that H and H_1 are the heights of the barometer reduced to 0° C. Thus if h is the measured height of the barometer at the lower station we have

$$\mathrm{H} = h\left(1 - \frac{t}{6500}\right).$$

If the height to be measured is not great, one observer is enough. For greater heights the ascent takes some time, and in the interval the pressure

may vary. Consequently in this case there must be two observers, one at each station, who make simultaneous observations.

Let us take the following example of the above formula :—Suppose that in latitude 65° N. at the lower of the two stations the height of the barometer were 30·025 inches, and the temperature of air and mercury 17°·32 C., while at the upper the height of the barometer was 28·230 inches, and the temperature of air and mercury was 10°·55 C. Determine the height of the upper station above the lower.

(1) Find H and H_1 : viz.

$$H = 30{\cdot}025\left(1 - \frac{17{\cdot}32}{6500}\right) = 29{\cdot}945$$

$$H_1 = 28{\cdot}230\left(1 - \frac{10{\cdot}55}{6500}\right) = 28{\cdot}184.$$

$$\text{Hence } \log\frac{H}{H_1} = 1{\cdot}4763243 - 1{\cdot}4500026 = 0{\cdot}0263217.$$

(2) Find $1 + \dfrac{2(T+T_1)}{1000}$ viz. 1·05574.

(3) Find $1 + 0{\cdot}00256 \cos 2\phi$.

Since $0{\cdot}00256 \cos 130° = -0{\cdot}00256 \cos 50° = -0{\cdot}001645$

therefore $1 + 0{\cdot}00256 \cos 2\phi = -0{\cdot}998355$.

Hence the required height in feet equals

$$60346 \times 0{\cdot}998355 \times 1{\cdot}05574 \times 0{\cdot}0063217 = 1674$$

It may be easily proved that if H and H_1 do not greatly differ, the Napierian logarithm of $\frac{H}{H_1}$ equals $2\,\frac{H - H_1}{H + H_1}$. If for instance H equals 30 inches, and H_1 equals 29 inches, the resulting error would not exceed the $\frac{1}{8400}$ part of the whole. Accordingly for heights not exceeding 2000 ft. we may without much error use the formula,

$$X \text{ (in feet)} = 52500\left(1 + \frac{2(T+T_1)}{1000}\right) \times \frac{H - H_1}{H + H_1}.$$

173. **Ruhlmann's observations.**—The results obtained for the difference in height of places by using the above formula often differ from the true heights as measured trigonometrically, to an extent which cannot be ascribed to errors in observation. The numbers thus found for the heights of places are influenced by the time of day, and also by the season of year, at which they are made. Ruhlmann has investigated the cause of this discrepancy by a series of direct barometric and thermometric observations made at two different stations in Saxony, and also by a comparison of the continuous series of observations made at Geneva and on the St. Bernard.

Ruhlmann has ascertained thus that the cause of the discrepancy is to be found in the fact that the mean of the temperatures indicated by the thermometer at the two stations is not an accurate measure of the actual mean temperature of the column of air between the two stations, a condition which is assumed in the above formula. The variations in the temperature

of the column of air are not of the same extent as those indicated by the thermometer, nor do they follow them so rapidly; they drag after them as it were. If the mean monthly temperatures at the two fixed stations are introduced into the formula, they give in winter heights which are somewhat too low, and in summer such as are too high. The results obtained by introducing the mean yearly temperature of the two stations are very near the true ones.

This influence of temperature is most perceptible in individual observations of low heights. Thus, using the observed temperatures in the barometric formula, the error in height of the Uetliberg above Zurich (about 1,700 feet) was found to be $\frac{1}{23}$ of the total, while the height of the St. Bernard above Geneva was found within $\frac{1}{135}$ of the true height.

The reason the thermometers do not indicate the true temperature of the air is undoubtedly that they are too much influenced by radiation from the earth and surrounding bodies. The earth is highly absorbent, and becomes rapidly heated under the influence of the sun's rays, and becomes as rapidly cooled at night; the air, as a very diathermanous body, is but little heated by the sun's rays, and on the contrary is little cooled by radiation during the night.

CHAPTER II.

MEASUREMENT OF THE ELASTIC FORCE OF GASES.

174. **Boyle's law.**—The law of the compressibility of gases was discovered by Boyle in 1662, and afterwards independently by Mariotte in 1679. It is in England commonly called 'Boyle's law,' and, on the Continent, 'Mariotte's law.' It is as follows :—

The temperature remaining the same, the volume of a given quantity of gas is inversely as the pressure which it bears.

This law may be verified by means of an apparatus devised by Boyle (fig. 141). It consists of a long glass tube fixed to a vertical support; it is open at the upper part, and the other end, which is bent into a short vertical leg, is closed. On the shorter leg there is a scale, which indicates equal capacities; the scale against the long leg gives the heights. The zero of both scales is in the same horizontal line.

A small quantity of mercury is poured into the tube, so that its level in both branches is at zero, which is effected without much difficulty after a few trials (fig. 141). The air in the short leg is thus under the ordinary atmospheric pressure which is exerted through the open tube. Mercury is then poured into the longer tube until the volume of the air in the smaller tube is reduced to one-half; that is, until it is reduced from 10 to 5, as shown in fig. 142. If the height of the mercurial column, CA, be measured, it will be found exactly equal to the height of the barometer at the time of the experiment. The pressure of the column CA is therefore equal to an atmosphere which, with the atmospheric pressure acting on the surface of the column at C, makes two atmospheres. Accordingly, by doubling the pressure, the volume of the gas has been diminished to one-half.

If mercury be poured into the longer branch until the volume of the air is reduced to one-third its original volume, it will be found that the distance between the level of the two tubes is equal to two barometric columns. The pressure is now three atmospheres, while the volume is reduced to one-third. Dulong and Petit have verified the law for air up to 27 atmospheres, by means of an apparatus analogous to that which has been described.

The law also holds good in the case of pressures of less than one atmosphere. To establish this, mercury is poured into a graduated tube until it is about two-thirds full, the rest being air. It is then inverted in a deep trough M containing mercury (fig. 143), and lowered until the levels of the mercury inside and outside the tube are the same, and the volume AB noted. The tube is then raised, as represented in the figure, until the volume of air, AC, is double that of AB (fig. 144). The height of the mercury in the tube

above the mercury in the trough, CD, is then found to be exactly half the height of the barometric column. The air, whose volume is now doubled, is now only under the pressure of half an atmosphere; for it is the elastic force of this air which, added to the weight of the column CD, is equivalent to the atmospheric pressure. Hence the volume is inversely as the pressure.

In the experiment with Mariotte's tube, as the quantity of air remains the same, its density must obviously increase as its volume diminishes, and *vice*

Fig. 141.

Fig. 142.

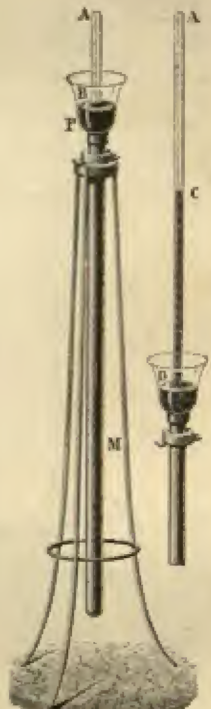

Fig. 143. Fig. 144.

versâ. The law may thus be enunciated: '*For the same temperature the density of a gas is proportional to its pressure.*' Hence as water is 773 times as heavy as air, under a pressure of 773 atmospheres, air would be as dense as water.

Boyle's law must not be understood to mean that gases of equal density have equal elastic force; different gases of various densities have the same tension when they are under the same pressure. A given volume of hydrogen under the ordinary atmospheric pressure has the same elastic force as the same volume of air, although the latter is 14 times as heavy as the former. Since, for the same volume, there are the same number of atoms in all gases,

the lighter atoms must possess a greater velocity in order to exert the same pressure as the same number of atoms of greater mass.

175. **Boyle's law is only approximately true.**—Until within the last few years Boyle's law was supposed to be absolutely true for all gases at all pressures, but Despretz obtained results incompatible with the law. He took two graduated glass tubes of the same length, and filled one with air and the other with the gas to be examined. These tubes were placed in the same mercury trough, and the whole apparatus immersed in a strong glass cylinder filled with water. By means of a piston moved by a screw which worked in a cap at the top of a cylinder, the liquid could be subjected to an increasing pressure, and it could be seen whether the compression of the two gases was the same or not. The apparatus resembled that used for examining the compressibility of liquids (fig. 63). In this manner Despretz found that carbonic acid, sulphuretted hydrogen, ammonia, and cyanogen are more compressible than air: hydrogen, which has the same compressibility as air up to 15 atmospheres, is then less compressible. From these experiments it was concluded that the law of Boyle was not general.

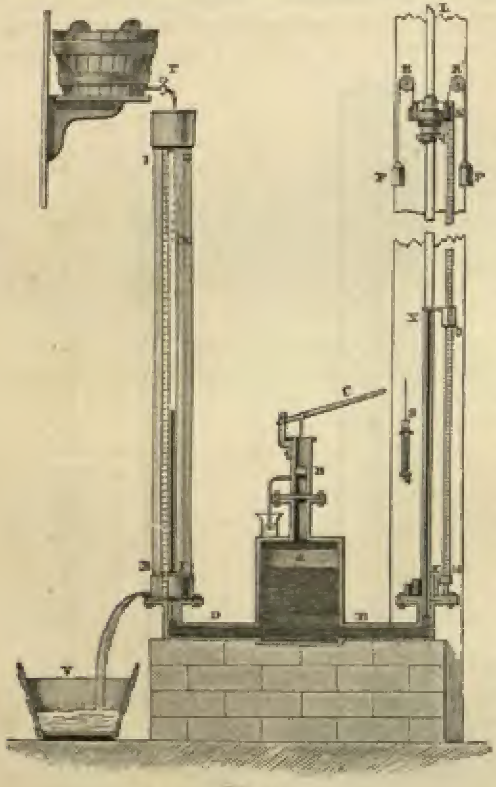
Fig. 145.

In some experiments on the elastic force of vapours, Dulong and Arago had occasion to test the accuracy of Boyle's law. The method adopted was exactly that of Mariotte, but the apparatus had gigantic dimensions.

The gas to be compressed was contained in a strong glass tube, GF (fig. 145), about six feet long and closed at the top, G. The pressure was produced by a column of mercury, which could be increased to a height of 65 feet, contained in a long vertical tube, KL, formed of a number of tubes firmly joined by good screws, so as to be perfectly tight.

The tubes KL and GF were hermetically fixed in a horizontal iron pipe

DE, which formed part of a mercurial reservoir, A. On the top of this reservoir there was a force pump, BC, by which mercury could be forced into the apparatus.

At the commencement of the experiment, the volume of the air in the manometer (177) was observed, and the initial pressure determined, by adding to the pressure of the atmosphere the height of the mercury in K above its level in H. If the level of the mercury in the manometer had been above the level in KL, it would have been necessary to subtract tne difference.

By means of the pump, water was injected into A. The mercury being then pressed by the water, rose in the tube GF, where it compressed the air, and in the tube KL, where it rose freely. It was only then necessary to measure the volume of the air in GF; the height of the mercury in KL above the level in GF, together with the pressure of the atmosphere, was the total pressure to which the gas was exposed. These were all the elements necessary for comparing different volumes and the corresponding temperatures. The tube GF was kept cold during the experiment by a stream of cold water.

The long tube was attached to a long mast by means of staples. The individual tubes were supported at the junction by cords, which passed round pulleys R and R′, and were kept stretched by small buckets, P, containing shot. In this manner, each of the thirteen tubes having been separately counterpoised, the whole column was perfectly free notwithstanding its weight.

Dulong and Arago experimented with pressures up to 27 atmospheres, and observed that the volume of air always diminished a little more than is required by Boyle's law. But as these differences were very small, they attributed them to errors of observation, and concluded that the law was perfectly exact, at any rate up to 27 atmospheres.

Regnault investigated the same subject with an apparatus resembling that of Dulong and Arago, but in which all the sources of error were taken into account, and the observations made with remarkable precision. He found that air does not exactly follow Boyle's law, but experiences a greater compressibility, which increases with the pressure; so that the difference between the calculated and the observed diminution of volume is greater in proportion as the pressure increases.

Regnault found that nitrogen was like air, but is less compressible. Carbonic acid exhibits considerable deviation from Boyle's law even under small pressures. Hydrogen also deviates from the law, but its compressibility diminishes with increased pressure.

Cailletet examined the compressibility of gases by a special method in which the pressure could be carried as high as 600 atmospheres. His results confirm those of Regnault as regards hydrogen; nitrogen was found to present the curious feature that towards 80 atmospheres it has *a maximum relative compressibility*; beyond this point it gradually becomes less compressible, its compressibility diminishing more rapidly than that of hydrogen. Carbonic acid deviates less from the law in proportion as the temperature is higher. This is also the case with other gases. And experiment shows that the deviation from the law is greater in proportion as the gas is nearer

its liquefying point; and, on the contrary, the farther a gas is from this point, the more closely does it follow the law. For gases which are the most difficult to liquefy, the deviations from the law are inconsiderable, and may be quite neglected in ordinary physical and chemical experiments, where the pressures are not great.

176. **Applications of Boyle's law.**—Observations on the volumes of gases are only comparable when made at the same pressure. Usually, therefore, in gas analyses, all measurements are reduced to the standard pressure of 760 millimetres, or 29·92 inches. This is easily done by Boyle's law, for, since the volumes are inversely as the pressures, $V : V' = P' : P$. Knowing the volume V at the pressure P, we can easily calculate its volume V′ at the given pressure P′, for

$$V'P' = VP; \qquad \text{that is, } V' = \frac{VP}{P'}.$$

Suppose a volume of gas to measure 340 cubic inches under a pressure of 535 mm., what will be its volume at the standard pressure, 760 mm.?

We have $$V = \frac{340 \times 535}{760} = 238 \text{ cubic inches.}$$

In like manner let it be asked, if D′ is the density of a gas when the barometer stands at H′ mm., what will be its density D at the same temperature when the barometer stands at H mm.?

Let M be the mass of the gas, V′ its volume in the first case, V its volume in the second. Therefore,

$$DV = M = D'V'$$

or, $$\frac{D}{D'} = \frac{V'}{V} = \frac{P}{P'} = \frac{H}{H'}$$

Thus, if H′ denote 760 mm., we have

$$\text{Density at } H' = (\text{Density at standard pressure}) \frac{H}{760}.$$

177. **Manometers.**—*Manometers* are instruments for measuring the tension of gases or vapours. In all such instruments the unit chosen is the pressure of one atmosphere or 30 inches of mercury at the standard temperature, which, as we have seen, is nearly 15 lbs. to the square inch.

178. **Open-air manometer.**—The *open-air manometer* consists of a bent glass tube BD (fig. 146), fastened to the bottom of a reservoir AC, of the same material, containing mercury, which is connected with the closed recipient containing the gas or vapour the pressure of which is to be measured. The whole is fixed on a long plank kept in a vertical position.

In graduating this manometer C is left open, and the number 1 marked at the level of the mercury, for this represents one atmosphere. From this point the numbers 2, 3, 4, 5, 6 are marked at each 30 inches, indicating so many atmospheres, since a column of mercury 30 inches represents a pressure of one atmosphere. The intervals from 1 to 2, and from 2 to 3, &c., are divided into tenths. C being then placed in connection with a boiler, for example, the mercury rises in the tube BD to a height which measures the

tension of the vapour. In the figure the manometer marks 2 atmospheres, which represents a height of 30 inches, plus the atmospheric pressure exerted at the top of the column through the aperture D.

This manometer is only used when the pressures do not exceed 5 to 6 atmospheres. Beyond this, the length of tube necessary makes it very inconvenient, and the following apparatus is commonly used.

179. **Manometer with compressed air.**—The *manometer with compressed air* is founded on Boyle's law: it consists of a glass tube closed at the top, and filled with dry air. It is firmly cemented in a small iron box containing mercury. By a tubulure, A, in the side (fig. 146), this box is connected with the closed vessel containing the gas or vapour whose tension is to be measured.

In the graduation of this manometer, the quantity of air contained in the tube is such that when the aperture A communicates freely with the atmosphere, the level of the mercury is the same in the tube and in the tubulure. Consequently, at this level, the number 1 is marked on the scale to which the tube is affixed. As the pressure acting through the tubulure A increases, the mercury

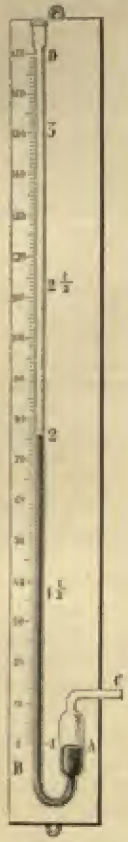

Fig. 146.

Fig. 147.

Fig. 148.

rises in the tube, until its weight, added to the tension of the compressed air, is equal to the external pressure. It would consequently be incorrect to mark two atmospheres in the middle of the tube; for since the volume of the air is reduced to one-half, its tension is equal to two atmospheres, and, together with the weight of the mercury raised in the tube, is there-

fore more than two atmospheres. The position of the number is a little below the middle, at such a height that the elastic force of the compressed air, together with the weight of the mercury in the tube, is equal to two atmospheres. The exact position of the numbers, 2, 3, 4, &c., on the manometer scale can only be determined by calculation. Sometimes this manometer is made of one glass tube (as represented in fig. 148). The principle is obviously the same.

180. **Volumometer.**—An interesting application of Boyle's law is met with in the *volumometer*. This consists of a glass tube with a cylinder G at the top (fig. 149), the edges of which are carefully ground, and which can be closed hermetically by means of a ground-glass plate D. The top being open, the tube is immersed until the level of the mercury inside and outside is the same: this is represented by the mark Z. The apparatus is then closed air-tight by the plate, and is raised until the mercury stands at a height h, above the level Q in the bath. The original volume of the enclosed air V, which was under the pressure of the atmosphere, is now increased to $V + v$, since the pressure has diminished by the height of the column of mercury h. Calling the pressure of the atmosphere at the time of observation b, we shall have $V : V + v = b - h : b$.

Fig. 149.

Placing now in the cylinder a body K whose volume x is unknown, the same operations are repeated, the tube is raised until the mercury again stands at the same mark as before, but its height above the bath is now different; a second reading, h_1, is obtained, and we have $(V - x) : (V - x) + v = b - h_1 : b$. Combining and reducing we get $x = (V + v)\left(1 - \frac{h}{h_1}\right)$. The volume $V + v$ is constant, and is determined numerically, once for all, by making the experiment with a substance of known volume, such as a glass bulb.

181. **Regnault's barometric manometer.**—For measuring pressures of less than one atmosphere, Regnault devised the following arrangement, which is a modification of his fixed barometer (fig. 138). In the same cistern dips a second tube a, of the same diameter, open at both ends, and provided at the top with a three-way cock, one of which is connected with an air-pump and the other with the space to be exhausted. The further the exhaustion is carried the higher the mercury rises in the tube a. The differences of level in the tubes b and a give the pressures. Hence, by measuring the height ab, by means of the cathetometer, the pressure in the space that is being exhausted is accurately given. This apparatus is also called the *differential barometer*.

182. **Aneroid barometer.**—This instrument derives its name from the circumstance that no liquid is used in its construction (ἀ, without, ὑγρὸς, moist). Fig. 150 represents one of the forms of these instruments, constructed by Casella; it consists of a cylindrical metal box, exhausted of air, the top of which is made of thin corrugated metal, so elastic that it readily yields to alterations in the pressure of the atmosphere.

When the pressure increases, the top is pressed inwards; when on the

contrary it decreases, the elasticity of the lid, aided by a spring, tends to move it in the opposite direction. These motions are transmitted by delicate multiplying levers to an index which moves on a scale. The instrument is graduated empirically by comparing its indications, under different pressures, with those of an ordinary mercurial barometer.

Fig. 150.

The aneroid has the advantage of being portable, and can be constructed of such delicacy as to indicate the difference in pressure between the height of an ordinary table and the ground. It is hence much used in determining heights in mountain ascents. But it is somewhat liable to get out of order, especially when it has been subjected to great variations of pressure; and its indications must from time to time be compared with those of a standard barometer.

The errors arising from the use of the aneroid are mainly due to the transmission of the motion of the lid by the multiplying arrangement. Goldsmid of Zurich devised a form in which the motion of the lid is directly observed.

Like that of other aneroids, the lid of the box *a* (fig. 151), in which the alterations of pressure are determined, is of fine corrugated sheet metal. To this is fixed a horizontal metal strip *b*, on the front end of which is a small square *e*, acting as index. This rises and falls with the movement of the lid, and indicates on a scale *ff'*, on the sides of the slit *dd'*, alterations in pressure of centimetres. To this strip a second and more delicate one, *c*, is fixed, on the front end of which is also fixed an index *e'*. Before making an observation, the horizontal line of this index is made to coincide with that of *e*; this is effected by means of a micrometer screw *m*, which is raised or lowered by the movable ring *h*; on the corresponding scale millimetres and tenths of a millimetre are read off. To do this the instrument is provided with a lens not represented in the figure. There is also a small thermometer *t*; from its indications a correction is made for temperatures according to an empirical scale specially constructed for each instrument.

Fig. 151.

183. **Laws of the mixture of gases.**—If a communication is opened between two closed vessels containing gases, they at once begin to mix,

whatever be their density, and in a longer or shorter time the mixture is complete, and will continue so, unless chemical action or some other extraneous cause intervene. The laws which govern the mixture of gases may be thus stated :—

I. *The mixture takes place rapidly and is homogeneous; that is, each portion of the mixture contains the two gases in the same proportion.*

II. *If the gases severally and the mixture have the same temperature, and if the gases severally and the mixture occupy the same volume, then the pressure on the unit of area exerted by the mixture will equal the sum of pressures on the unit of area exerted by the gases severally.*

From the second law a very convenient formula can be easily deduced.

Let $v_1, v_2, v_3 \ldots$ be the volumes of several gases under pressure of $p_1, p_2, p_3 \ldots$ respectively. Suppose these gases when mixed to have a volume V, under a pressure P, the temperatures being the same. By Boyle's law we know that v_1 will occupy a volume V under a pressure p_1' provided that

$$Vp_1' = v_1 p_1$$

Similarly

$$Vp_2' = v_2 p_2$$

and so on. But we learn from the above law that

$$P = p_1' + p_2' + \ldots$$

therefore

$$VP = v_1 p_1 + v_2 p_2 + v_3 p_3 + \ldots$$

It obviously follows that if the pressures are all the same, the volume of the mixture equals the sum of the separate volumes.

The first law was shown experimentally by Berthollet, by means of an apparatus represented in fig. 152. It consists of two glass globes provided with stopcocks, which can be screwed one on the other. The upper globe was filled with hydrogen, and the lower one with carbonic acid, which has 22 times the density of hydrogen. The globes having been fixed together were placed in the cellars of the Paris Observatory and the stopcocks then opened, the globe containing hydrogen being uppermost. Berthollet found after some time that the pressure had not changed, and that, in spite of the difference in density, the two gases had become uniformly mixed in the two globes. Experiments made in the same manner with other gases gave the same results, and it was found that the diffusion was more rapid in proportion as the difference between the densities was greater.

Fig. 152.

The second law may be demonstrated by passing into a graduated tube, over mercury, known volumes of gas at known pressures. The pressure and volume of the whole mixture are then measured, and found to be in accordance with the law.

Gaseous mixtures follow Boyle's law, like simple gases, as has been proved for air (174), which is a mixture of nitrogen and oxygen.

184. **Mixture of gases and liquids. Absorption of gases.**—Water and many liquids possess the property of absorbing gases. Under the same conditions of pressure and temperature a liquid does not absorb equal quantities of different gases. At the temperature 0°C. and pressure 760 mm. one volume of water dissolves the following volumes of gas :—

Nitrogen	0·020	Sulphuretted hydrogen . .	4·37
Oxygen	0·041	Sulphurous Acid . . .	79·79
Carbonic Acid . . .	1·79	Ammonia	1046·63

From the very great condensation, to which the latter correspond, it may be inferred that the gases are in the liquid state.

Gases are more soluble in alcohol ; thus at 0°C. alcohol dissolves 4·33 volumes of carbonic acid gas.

The whole subject of gas absorption has been investigated by Bunsen. The general laws are the following :—

I. *For the same gas, the same liquid, and the same temperature, the weight of gas absorbed is proportional to the pressure.* This may also be expressed by saying that at all pressures the volume dissolved is the same ; or that the density of the gas absorbed is in a constant relation with that of the external gas which is not absorbed.

Accordingly, when the pressure diminishes, the quantity of dissolved gas decreases. If a solution of gas be placed under the air-pump and a vacuum created, the gas obeys its expansive force and escapes with effervescence.

II. *The quantity of gas absorbed decreases with the temperature* ; that is to say, when the elastic force of the gas is greater. Thus at 15° water only absorbs 1·00 of carbonic acid.

III. *The quantity of gas which a liquid can dissolve is independent of the nature and of the quantity of other gases which it may already hold in solution.*

In every gaseous mixture each gas exercises the same pressure as it would if its volume occupied the whole space ; and the total pressure is equal to the sum of the individual pressures. When a liquid is in contact with a gaseous mixture, it absorbs a certain part of each gas, but less than it would if the whole space were occupied by each gas. The quantity of each gas dissolved is proportional to the pressure which the unabsorbed gas exercises alone. For instance, oxygen forms only about $\frac{1}{5}$ the quantity of air ; and water, under ordinary conditions, absorbs exactly the same quantity of oxygen as it would if the atmosphere were entirely formed of this gas under a pressure equal to $\frac{1}{5}$ that of the atmosphere.

CHAPTER III.

PRESSURE ON BODIES IN AIR. BALLOONS.

185. **Archimedes' principle applied to gases.**—The pressure exerted by gases, on bodies immersed in them, is transmitted equally in all directions, as has been shown by the experiment with the Magdeburg hemispheres. It therefore follows that all which has been said about the equilibrium of bodies in liquids applies to bodies in air; they lose a part of their weight equal to that of the air which they displace.

Fig. 153.

The loss of weight in air is demonstrated by means of the *baroscope*, which consists of a scalebeam, at one of whose extremities a small leaden weight is supported, and at the other there is a hollow copper sphere (fig. 153). In the air they exactly balance one another; but when they are placed under the receiver of the air-pump, and a vacuum is produced, the sphere sinks, thereby showing that in reality it is heavier than the small leaden weight. Before the air is exhausted each body is buoyed up by the weight of the air which it displaces. But as the sphere is much the larger of the two, its weight undergoes most apparent diminution, and thus, though in reality the heavier body, it is balanced by the small leaden weight. It may be proved by means of the same apparatus that this loss is equal to the weight of the displaced air. Suppose the volume of the sphere is 10 cubic inches. The weight of this volume of air is 3·1 grains. If now this weight be added to the leaden weight, it will overbalance the sphere in air, but will exactly balance it in vacuo.

The principle of Archimedes is true for bodies in air; all that has been said about bodies immersed in liquids applies to them; that is, that when a body is heavier than air, it will sink, owing to the excess of its weight over the buoyancy. If it is as heavy as air, its weight will exactly counterbalance the buoyancy, and the body will float in the atmosphere. If the body is lighter than air, the buoyancy of the air will prevail, and the body will rise in the atmosphere until it reaches a layer of the same density as its own. The force of the ascent is equal to the excess of the buoyancy over the

weight of the body. This is the reason why smoke, vapours, clouds, and air balloons rise in the air.

AIR BALLOONS.

186. **Air balloons.**—*Air balloons* are hollow spheres made of some light impermeable material, which, when filled with heated air, with hydrogen gas, or with coal gas, rise in the air by virtue of their relative lightness.

They were invented by the brothers Mongolfier of Annonay, and the first experiment was made at that place in June 1783. Their balloon was a sphere of forty yards in circumference, and weighed 500 pounds. At the lower part there was an aperture, and a sort of boat was suspended, in which fire was lighted to heat the internal air. The balloon rose to a height of 2,200 yards, and then descended without any accident.

Charles, a professor of physics in Paris, substituted hydrogen for hot air. He himself ascended in a balloon of this kind in December 1783. The use of hot-air balloons was entirely given up in consequence of the serious accidents to which they were liable.

Since then the art of ballooning has been greatly extended, and many ascents have been made. That which Gay-Lussac made in 1804 was the most remarkable for the facts with which it has enriched science, and for the height which he attained—23,000 feet above the sea level. At this height the barometer descended to 12·6 inches, and the thermometer which was 31° C. on the ground was 9 degrees below zero.

In these high regions, the dryness was such on the day of Gay-Lussac's ascent, that hygrometric substances, such as paper, parchment, &c., became dried and crumpled as if they had been placed near the fire. The respiration and circulation of the blood were accelerated in consequence of the great rarefaction of the air. Gay-Lussac's pulse made 120 pulsations in a minute instead of 66, the normal number. At this great height the sky had a very dark blue tint, and an absolute silence prevailed.

One of the most remarkable of recent ascents was made by Mr. Glaisher and Mr. Coxwell, in a large balloon belonging to the latter. This was filled with 90,000 cubic feet of coal gas (sp. gr. 0·37 to 0·33); the weight of the load was 600 pounds. The ascent took place at 1 P.M. on September 5, 1861; at 1.28 they had reached a height of 15,750 feet, and in eleven minutes after a height of 21,000 feet, the temperature being $-10·4°$; at 1.50 they were at 26,200 feet, with the thermometer at $-15·2°$. At 1.52 the height attained was 29,000 feet, and the temperature $-16°$ C. At this height the rarefaction of the air was so great, and the cold so intense, that Mr. Glaisher fainted, and could no longer observe. According to an approximate estimation the lowest barometric height they attained was 7 inches, which would correspond to an elevation of 36,000 to 37,000 feet.

187. **Construction and management of balloons.**—A balloon is made of long bands of silk sewed together and covered with caoutchouc varnish, which renders it air-tight. At the top there is a safety valve closed by a spring, which the aëronaut can open at pleasure by means of a cord. A light wickerwork boat is suspended by means of cords to a network, which entirely covers the balloon.

A balloon of the ordinary dimensions, which can carry three persons, is about 16 yards high, 12 yards in diameter, and its volume, when it is quite full, is about 680 cubic yards. The balloon itself weighs 200 pounds; the accessories, such as the rope and boat, 100 pounds.

Fig. 154.

The balloon is filled either with hydrogen or with coal gas. Although the latter is heavier than the former, it is generally preferred, because it is cheaper and more easily obtained. It is passed into the balloon from the gas reservoir by means of a flexible tube. It is important not to fill the balloon quite full, for the atmospheric pressure diminishes as it rises (fig. 154), and the gas inside, expanding in consequence of its elastic force, tends to burst it. It is sufficient for the ascent if the weight of the displaced air exceeds that of the balloon by 8 or 10 pounds. And this force remains constant so long as the balloon is not quite distended by the dilatation of the air in the interior. If the atmospheric pressure, for example, has diminished to one-half, the gas in the balloon, according to Boyle's law, has doubled its volume. The volume of the air displaced is therefore twice as great; but since its density has become only one-half, the weight and consequently the upward buoyancy are the same. When once the balloon is completely dilated, if it continues to rise, the force of the ascent decreases, for the volume of the displaced air remains the same, but its density diminishes, and a time arrives at which the buoyancy is equal to the weight of the balloon. The balloon can now only take a horizontal direction, carried by the currents of air which prevail in the atmosphere. The aëronaut knows by the barometer whether he is ascending or descending, and by the same means he determines the height which he has reached. A long flag fixed to the boat would indicate, by the position it takes either above or below, whether the balloon is descending or ascending.

When the aëronaut wishes to descend, he opens the valve at the top of the balloon by means of the cord, which allows gas to escape, and the balloon sinks. If he wants to descend more slowly, or to rise again, he empties out bags of sand, of which there is an ample supply in the car. The descent is facilitated by means of a grappling iron fixed to the boat. When

once this is fixed to any obstacle, the balloon is lowered by pulling the cord.

The only practical applications which air balloons have hitherto had have been in military reconnoitring. At the battle of Fleurus, in 1794, a captive balloon—that is, one held by a rope—was used, in which there was an observer who reported the movements of the enemy by means of signals. At the battle of Solferino the movements and dispositions of the Austrian troops were watched by a captive balloon; and in the war in America balloons were frequently used, while their importance during the siege of Paris is fresh in all memories. The whole subject of military ballooning was treated in two papers by Captain Grover and by Captain Beaumont, in a volume of the Professional Papers of the Royal Engineers; and experiments are now in progress, at Woolwich and at Aldershot, with a view of ascertaining the most practicable means of inflating balloons and the best form and equipment for service in the field. It has been proposed to use captive balloons for observations on the changes of temperature in the air, &c. Air balloons can only be truly useful when they can be guided, and as yet all attempts made with this view have completely failed. There is no other course at present than to rise in the air until there is a current which has more or less the desired direction. Unfortunately the currents in the higher regions of the atmosphere are variable and irregular.

188. **Parachute.**—The object of the parachute is to allow the aëronaut to leave the balloon, by giving him the means of lessening the rapidity of his descent. It consists of a large circular piece of cloth (fig. 155), about 16 feet in diameter, and which by the resistance of the air spreads out like a gigantic umbrella. In the centre there is an aperture, through which the air compressed by the rapidity of the descent makes its escape; for otherwise oscillations might be produced, which, when communicated to the boat, would be dangerous.

In fig. 154 there is a parachute attached to the network of the balloon by means of a cord which passes round a pulley, and is fixed at the other end to the boat. When the cord is cut the parachute sinks, at first very rapidly, but more slowly as it becomes distended, as represented in the figure.

Fig. 155.

189. **Calculation of the weight which a balloon can raise.**—To calculate the weight which can be raised by a balloon of given dimen-

sions, let us suppose it perfectly spherical, and premise that the formulæ which express the volume and the superficies in terms of the radius are $V = \frac{4\pi R^3}{3}$ $S = 4\pi R^2$; π being the ratio of the circumference to the diameter. The radius R being measured in feet, let p be, in pounds, the weight of a square foot of the material of which the balloon is constructed; let P be the weight of the car and the accessories, a the weight in pounds of a cubic foot of air at zero, and under the pressure $0{\cdot}76^m$, and a' the weight of the same volume, under the same conditions, of the gas with which the balloon is inflated (149). Then the total weight of the envelope in pounds will be $4\pi R^2 p$; that of the gas will be $\frac{4\pi R^3 a'}{3}$, and that of the displaced air $\frac{4\pi R^3 a}{3}$. If X be the weight which the balloon can support, we have

$$X = \frac{4\pi R^3 a}{3} - \frac{4\pi R^3 a'}{3} - 4\pi R^2 p - P.$$

Whence

$$X = \frac{4\pi R^3}{3}(a - a') - 4\pi R^2 p - P.$$

But as we have before seen (186), in order that the balloon may rise, the weights must be less by 8 or 10 pounds than that given by this equation.

CHAPTER IV.

APPARATUS WHICH DEPEND ON THE PROPERTIES OF AIR.

190. **Air-pump.**—The air-pump is an instrument by which a vacuum can be produced in a given space, or rather by which air can be greatly rarefied, for an absolute vacuum cannot be produced by its means. It was invented by Otto von Guericke in 1650, a few years after the invention of the barometer.

The air-pump, as now usually constructed, may be described as follows. In fig. 156, which shows the general arrangement, E is the *receiver*, in which the vacuum is to be produced. It is a bell glass resting on a plate D, of thick glass ground perfectly smooth. In the centre of D, at C, there is an opening by which a communication is made between the interior of the receiver and of the cylinders P, P. This communication is effected by a tube or pipe passing through the body of the plate A, and then branching off at right angles, as shown by *Keo Kes*, in fig. 157, which represents a horizontal section of the machine. In the cylinders—which are commonly of glass and which are firmly cemented to the plate A—are two pistons, P and Q, moving air-tight. Each piston is moved by a rack, working with a pinion, H, turning by a handle, M. This is shown more plainly in fig. 158, which represents a vertical section of the machine through the cylinders; here H is the pinion, and MN the handle. When M is forced down one piston is raised, and the other depressed.

Fig. 156.

When M's action is reversed, the former piston is depressed, and the latter raised.

The action of the machine is this. Each cylinder is fitted with a valve so contrived that, when its piston is raised, communication is opened between the cylinder and the receiver: when it is depressed the communication is closed. Now if P were simply raised, a vacuum would be formed below P; but as a communication is opened with the receiver E, the air in E expands so as to fill both the receiver and the cylinder. As soon as the piston begins to descend, the communication is closed, and none of the air in the cylinder returns to the receiver, but, by means of properly constructed

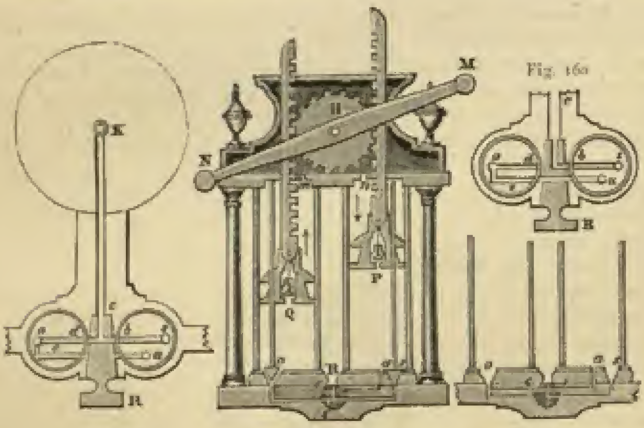

Fig. 157. Fig. 158. Fig. 159.

valves, escapes into the atmosphere. Consequently the rarefaction which the air in the receiver has undergone is permanent. By the next stroke a further rarefaction is produced: and so on, at each succeeding stroke.

It is clear that when the rarefaction has proceeded to a considerable extent, the atmospheric pressure on the top of P will be very great, but it will be very nearly balanced by the atmospheric pressure on the top of the other piston. Consequently the experimenter will have to overcome only the difference of the two pressures. This is the reason why two cylinders are employed.

To explain the action of the valves we must go into particulars. The general arrangement of the interior of the cylinders is shown in fig. 158. Fig 161 shows the section of a piston in detail. The piston is formed of two brass discs (X and V), screwed to one another, and compressing between them a series of leather discs Z, whose diameters are slightly greater than those of the brass discs. The leather is thoroughly saturated with oil, so as to slide air-tight, though with but little friction, within the cylinder. To the centre of the upper disc is screwed a piece, B, to which the rack H is riveted. The piece B is pierced, so as to put the interior of the cylinder into communication with the external air. This communication is closed by a valve *t*, held down by a delicate spring *r*. When the piston is moved downward

the air below the piston is compressed until it forces up *t* and escapes. The instant the action is reversed, the valve *t* falls, and is held down by the spring, and by the pressure of the external air, which is thereby kept from coming in. The communication between the cylinder below the piston and the receiver is opened and closed by the valve marked *o* in fig. 158, and *sg* in fig. 161. The rod *sg* passing through the piston is held by friction, and is raised with it; but is kept from being lifted through more than a very small distance by the top of the cylinder, while the piston, in continuing its upward motion, slides over *sg*. When the piston descends it brings the valve with it, which at once cuts off the communication between the cylinder and the receiver.

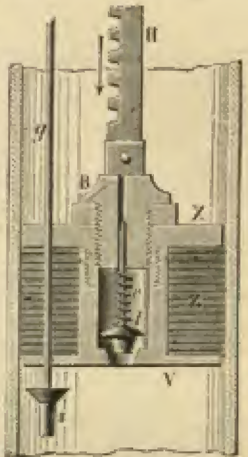

Fig. 161.

191. **Air-pump gauge.**—When the pump has been worked some time, the pressure in the receiver is indicated by the difference of level of the mercury in the two legs of a glass tube bent like a syphon, one of which is opened, and the other closed like the barometer. This little apparatus, which is called the *gauge*, is fixed to an upright scale, and placed under a small bell jar, which communicates with the receiver E by a stopcock, A, inserted in the tube leading from the orifice C to the cylinders, fig. 156.

Before commencing to exhaust the air in the receiver, its elastic force exceeds the weight of the column of mercury, which is in the closed branch and which consequently remains full. But as the pump is worked, the elastic force soon diminishes, and is unable to support the weight of the mercury, which sinks and tends to stand at the same level in both legs. If an absolute vacuum could be produced, they would be exactly on the same level, for there would be no pressure either on the one side or the other. But with the very best machines the level is always about a thirtieth of an inch higher in the closed branch, which indicates that the vacuum is not absolute, for the elastic force of the residue is equal to the pressure of a column of mercury of that height.

Theoretically an absolute vacuum is impossible; for, since the volume of each cylinder is, say, $\frac{1}{10}$ that of the receiver, only $\frac{1}{11}$ of the air in the receiver is extracted at each stroke of the piston, and consequently it is impossible to exhaust all the air which it contains. The theoretical degree of exhaustion after a given number of strokes is easily calculated as follows:—Let A denote the volume of the receiver, including in that term the pipe; B the volume of the cylinder between the highest and lowest positions of the piston; and assume for the sake of distinctness that there is only one cylinder; then the air which occupied A before the piston is lifted occupies A + B after it is lifted, and consequently if D_1 is the density at the end of the first stroke and D the orginal density, we must have

$$D_1 = D\frac{A}{A+B}$$

If D_2 is the density at the end of the second stroke, we have for just the same reason

$$D_2 = D_1 \frac{A}{A+B} = D\left(\frac{A}{A+B}\right)^2$$

Now this reasoning will apply to n strokes;

consequently $$D_n = D\left(\frac{A}{A+B}\right)^n$$

If there are two equal cylinders, the same formula holds; but in this case, in counting n, upstrokes and downstrokes equally reckon as *one*.

It is obvious that the exhaustion is never complete, since D can be zero only when n is infinite. However, no very great number of strokes is required to render the exhaustion virtually complete, even if A is several times greater than B. Thus if A = 10 B, a hundred strokes will reduce the density from D to 0·0004 D; that is, if the initial pressure is 30 in., the pressure at the end of 100 strokes is 0·012 of an inch.

Practically, however, a limit is placed on the rarefaction that can be produced by any given air-pump; for, as we have seen, the air becomes ultimately so rarefied that, when the pistons are at the bottom of the cylinder, its elastic force cannot overcome the pressure on the valves in the inside of the piston; they therefore do not open, and there is no further action of the pump.

192. **Doubly-exhausting stopcock.**—Babinet invented an improved stopcock, by which the exhaustion of the air can be carried to a very high degree. This stopcock is placed in the fork of the pipe leading from the receiver to the two cylinders; it is perforated by several channels, which are successively used by turning it into two different positions. Fig. 157 represents a horizontal section of the stopcock R, in such a position that, by its central opening and two lateral openings, it forms a communication between the orifice K of the plate, and the two valves o and s. The machine then works as has been described. In fig. 160 the stopcock has been turned a quarter, and the transversal channel ab, which was horizontal in fig. 157, is now vertical, and its extremities are closed by the side of the hole in which the stopcock works. But a second channel, which was closed before, and which has taken the place of the first, now places the right cylinder *alone* in communication with the receiver by the channel cbs (fig. 160), and it further connects the right with the left cylinder by a channel aeo (fig. 160), or $aico$ (fig. 158). This channel passes from a central opening a, placed at the base of the right cylinder, across the stopcock to the valve, o, of the other cylinder, as represented in figs. 159 and 160; but this channel is closed by the stopcock when it is in its first position, as is seen in figs. 157 and 158.

The right piston in rising exhausts the air of the receiver, but when it descends the exhausted air is driven into the left cylinder through the orifice a, the channel io, and the valve o (fig. 159), which is open. When the same piston rises, that of the left sinks; but the air which is above it does not return into the right cylinder, because the valve o is now closed. As the right cylinder continues to exhaust the air in the receiver,

and to force it into the left cylinder, the air accumulates here, and ultimately acquires sufficient tension to raise the valve of the piston Q, which was impossible before the stopcock was turned, for it is only when the valves in the piston no longer open, that a quarter of a turn is given to the stopcock.

193. **Bianchi's air-pump.**—Bianchi invented an air-pump which has several advantages. It is made entirely of iron, and it has only one cylinder,

Fig. 162.

which oscillates on a horizontal axis fixed at its base as seen in fig. 162. A horizontal shaft, with heavy fly-wheel, V, works in a frame, and is turned by a handle, M. A crank, *m*, which is joined to the top of the piston-rod, is fixed to the same shaft, and consequently at every revolution of the wheel the cylinder makes two oscillations.

In some cases, as in that shown in the figure, the crank and the fly-wheel are on parallel axes connected by a pair of cog-wheels. The modification in the action produced by this arrangement is as follows:—If the cog-wheel on the former axis has twice as many teeth as that on the latter axis, the pressure which raises the piston is doubled; an advantage which is counterbalanced by the inconvenience that now the piston will make one oscillation for one revolution of the fly-wheel.

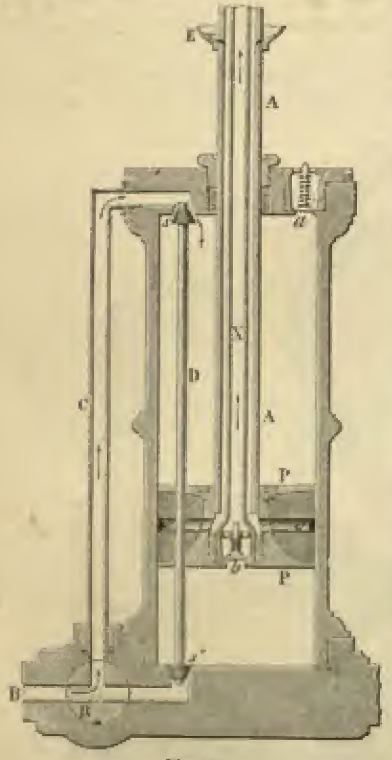

Fig. 163.

The machine is double acting; that is, the piston PP (fig. 163) produces a vacuum, both in ascending and descending. This is effected by the following arrangements:—In the piston there is a valve, *b*, opening upwards as in the ordinary machine. The piston rod AA is hollow, and in the inside there is a copper tube, X, by which the air makes its escape through the valve *b*. At the top of the cylinder there is a second valve, *a*, opening upwards. An iron rod, D, works with gentle friction in the piston, and terminates at its ends in two conical valves, *s* and *s′*, which fit into the openings of the tube BB leading to the receiver.

Let us suppose the piston descends. The valve *s′* is then closed, and, the valve *s* being open, the air of the receiver passes in the space above the piston, while the air in the space below the piston undergoes compression, and, raising the valve, escapes by the tube X, which communicates with the atmosphere. When the piston ascends, the exhaustion takes place through *s′*, and the valve *s* being closed, the compressed air escapes by the valve *a*.

The machine has a stopcock for double exhaustion, similar to that already described (192). It is also oiled in an ingenious manner. A cup, E, round the rod is filled with oil, which passes into the annular space between the rod AA and the tube X; it passes then into a tube *oo*, in the piston, and, forced by the atmospheric pressure, is uniformly distributed on the surface of the piston.

The apparatus, being of iron, may be made of much greater dimensions than the ordinary air-pump. A vacuum can also be produced with it in far less time and in apparatus of greater size than usual.

194. **Deleuil's air-pump.**—In this air-pump the main peculiarity is its

piston, which is of considerable length and consists of a series of accurately constructed metal discs bolted together. This works easily and smoothly in the barrel, and no packing or lubricator is used; or rather the lubricator is the air in the space between the piston and the barrel. The internal friction of the air in this narrow space is so great that the rate at which it leaks into the barrel is far inferior to the rate at which the pump is exhausting air from the receiver. And Clerk Maxwell has shown that the internal friction is not diminished even when its density is greatly reduced. Hence the pump works very satisfactorily up to a considerable degree of exhaustion—to a millimetre of mercury, for instance.

195. **Sprengel's air-pump.**—Sprengel has devised a form of air-pump which depends on the principle of converting the space to be exhausted into a Torricellian vacuum.

If an aperture be made in the top of a barometer tube, the mercury sinks and draws in air; if the experiment be so arranged as to allow air to enter along with mercury, and if the supply of air be limited while that of mercury is unlimited, the air will be carried away and a vacuum produced. The following is the simplest form of the apparatus in which this action is realised. In fig. 164 *cd* is a glass tube longer than a barometer, open at both ends, and connected, by means of india-rubber tubing, with a funnel, A, filled with mercury and supported by a stand. Mercury is allowed to fall in this tube at a rate regulated by a clamp at *c*; the lower end of the tube *cd* fits in the flask B, which has a spout at the side a little higher than the lower end of *cd*; the upper part has a branch at *x* to which a receiver R can be tightly fixed. When the clamp at *c* is opened, the first portions of mercury which run out close the tube and prevent air from entering below. As the mercury is allowed to run down, the exhaustion begins, and the whole length of the tube from *x* to *d* is filled with cylinders of air and mercury having a downward motion. Air and mercury escape through the spout of the bulb B which is above the basin A, where the mercury is collected. It is poured back from time to time into the funnel A, to be repassed through the tube until the exhaustion is complete. As this point is approached, the enclosed air between the mercury

Fig. 164.

cylinders is seen to diminish, until the lower part of *cd* forms a continuous column of mercury about 30 inches high. Towards this stage of the process a noise is heard like that of a water-hammer when shaken; the operation is completed when the column of mercury encloses no air, and a drop of mercury falls on the top of the column without enclosing the slightest air-bubble. The height of the column then represents the height of the column of mercury in the barometer; in other words it is a barometer whose Torricellian vacuum is the receiver R. This apparatus has been used with great success in experiments in which a very complete exhaustion is required, as in the preparation of Geissler's tubes. (See Book X. Chapter VI.) It may be advantageously combined with an exhausting syringe, which first removes the greater part of the air, the exhaustion being then completed as above.

The most perfect vacua are obtained by absorbing the residual gas, after the exhaustion has been pushed as far as possible, either mechanically, or by some substance with which it combines chemically. Thus Dewar has produced a vacuum which he estimates at $\frac{1}{550}$ of a millimetre by heating charcoal to redness, in a vessel from which air had been exhausted by the Sprengel pump, and then allowing it to cool. Finkener filled a vessel with oxygen, then exhausted as far as possible, and finally heated to redness some copper contained in the vessel. This absorbed the minute quantity of gas left, with the formation of cupric oxide. In some of his experiments Crookes obtained by chemical means a vacuum of $\frac{1}{13000}$ of a millimetre. In these highly rarefied gases the pressure is so low that it is very difficult to measure minute differences. For such cases McLeod has devised a very valuable method, the principle of which is to condense a measured volume of the highly rarefied gas to a much smaller volume, and then to measure its pressure under the new conditions.

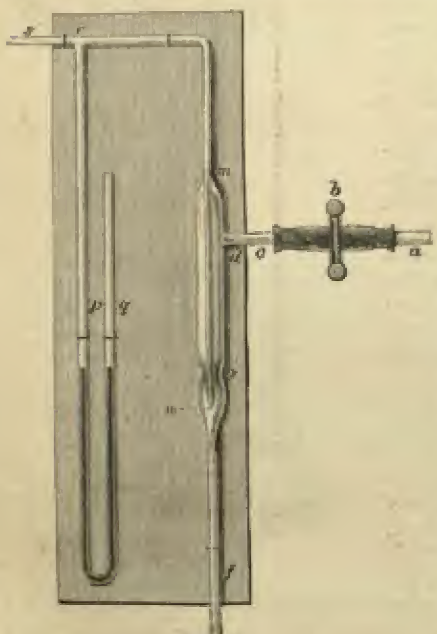

Fig. 165.

196. **Bunsen's filter pump.**—This is a very convenient arrangement for producing a vacuum in cases where a good supply of water is available, as in laboratories. Its principle is the same as that of Sprengel's pump. A composition tube *a* (fig. 165), connected with the service-pipe of a water-supply, is joined by means of a caoutchouc tube to a glass tube *cdf*, to which is attached at *f* a leaden tube

about 10 to 12 yards long. The tube *sr* is connected with the space to be exhausted. The water enters by *a*, and in falling down the tube carries with it air from the space to be exhausted. The supply of water, and therewith the rate of exhaustion, can be regulated by the stopcock *b*; the bent tube, *pq*, which contains mercury, measures the degree of exhaustion, which may be reduced to a pressure of 10 to 15 millimetres.

197. **Aspirating action of currents of air.**—When a jet of liquid or of a gas passes through air it carries the surrounding air along with it; fresh air rushes in to supply its place, comes also in contact with the jet, and is in like manner carried away. Thus, then, there is a continual rarefaction of the air around the jet, in consequence of which it exerts an aspiratory action.

This phenomenon may be well illustrated by means of an apparatus represented in fig. 166, the analogy of which to the experiment described (213) will be at once evident. It consists of a wide glass tube in the two ends of which are fitted two small tubes *nd* and B; in the bottom is a manometer tube containing a coloured liquid. On blowing through the narrow tube the liquid at *o* is seen to rise. If, on the contrary, the wide tube be blown into, a depression is produced at *o*.

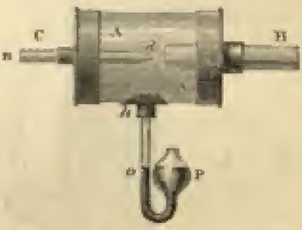

Fig. 166.

To this class of phenomena belongs the following experiment, which is a simple modification by Faraday of one originally described by Clement and Desormes. Holding one hand horizontal, the palm downwards and the fingers closed, you blow through the space between the index and middle finger. If a piece of light paper, of 2 or 3 square inches, is held against the aperture, it does not fall as long as the blowing continues.

The old *water-bellows* still used in mountainous places where there is a continuous fall is a further application of the principle. Water falling from a reservoir down a narrow tube divides and carries air along with it; and if there are apertures in the side through which air can enter, this also is carried along, and becomes accumulated in a reservoir placed below, from which by means of a lateral tube it can be directed into the hearth of a forge.

By the *locomotive steampipe* a jet of steam entering the chimney of the locomotive carries the air away, so that fresh air must arrive through the fire and thus the draught be kept up. In *Giffard's injector* water is pumped by means of a jet of steam into the boiler of a steam-engine.

198. **Morren's mercury pump.**—Figs. 167 and 168 represent a mercurial air-pump, which is an improvement by Alvergniat of a form devised by Morren.

It consists of two reservoirs, A and B, figs. 167 and 168, connected by a barometer tube T and a long caoutchouc tube C. The reservoir B and the tube T are fixed to a vertical support A, which is movable and open, and can be alternately raised and lowered through a distance of nearly four feet. This is effected by means of a long wire rope, which is fixed at one end to

the reservoir A, and passes over two pulleys, *a* and *b*, the latter of which is turned by a handle. Above the reservoir B is a three-way cock *n*; to this is attached a tube *d*, for exhaustion, and on the left is an ordinary stopcock *m*, which communicates with a reservoir of mercury *v*, and with the air. The exhausting tube *d* is not in direct communication with the receiver to be exhausted; it is first connected with a reservoir *o*, partially filled with sulphuric acid, and designed to dry the gases which enter the apparatus. A caout-

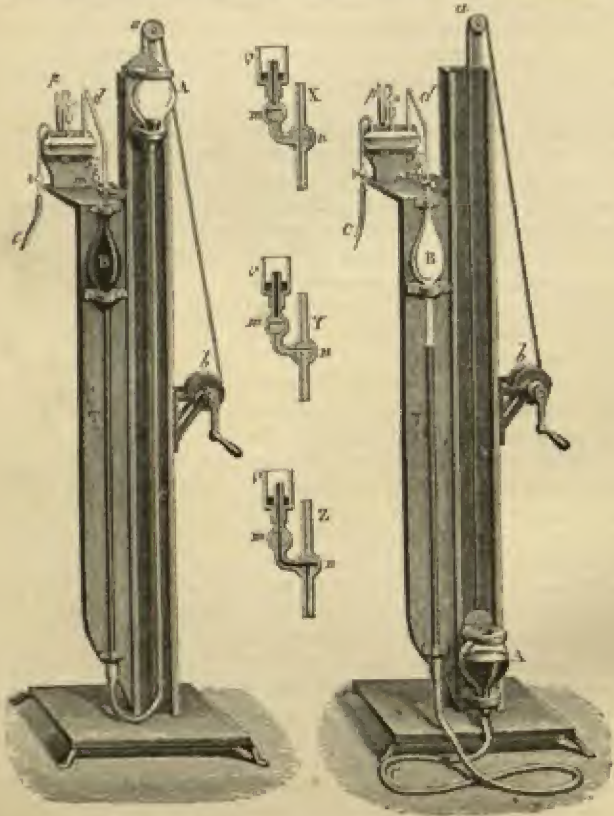

Fig. 167. Fig. 168.

chouc tube, *c*, makes communication with the receiver which is to be exhausted. On the reservoir *o* is a small mercury manometer *p*.

These details being understood, suppose the reservoir A at the top of its course (fig. 167), the stopcock *m* open, and the stopcock *n* turned as seen in Z; the caoutchouc tube C, the tube T, the reservoir B, and the tube above are filled with mercury as far as *v*; closing then the stopcock *m*, and lowering the reservoir A (fig. 168), the mercury sinks in the reservoir B, and in the

tube T, until the difference of levels in the two tubes is equal to the barometric height, and there is a vacuum in the reservoir B. Turning now the stopcock *n*, as shown in figure X, the gas from the space to be exhausted passes into the barometric chamber B, by the tubes *c* and *d*, and the level again sinks in the tube T. The stopcocks are now replaced in the first position (fig. Z), and the reservoir A is again lifted, the excess of pressure of mercury in the caoutchouc tube expels through the stopcocks *n* and *m* the gas which had passed into the chamber B, and if a few droplets of mercury are carried along with them they are collected in the vessel *v*. The process is repeated until the mercury is virtually at the same level in both legs.

Like Sprengel's pump, this is very slow in its working, and, like it, is best employed in completing the exhaustion of a space which has already been partially rarefied; for a vacuum of $\frac{1}{10}$ of a millimetre may be obtained by its means.

199. **Condensing pump.**—The condensing pump is an apparatus for compressing air, or any other gas. The form usually adopted is the following:—In a cylinder, A, of small diameter (fig. 170), there is a solid piston, the rod of which is moved by the hand. The cylinder is provided with a screw which fits into the receiver K. Fig. 169 shows the arrangement of the valves, which are so constructed that the lateral valve *o* opens from the outside, and the lower valve *s* from the inside.

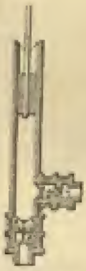
Fig. 169.

Fig. 170.

When the piston descends, the valve *o* closes, and the elastic force of the compressed air opens the valve *s*, which thus allows the compressed air to pass into the receiver. When the piston ascends, *s* closes and *o* opens, and permits the entrance of fresh air, which in turn becomes compressed by the descent of the piston, and so on.

This apparatus is chiefly used for charging liquids with gases. For this purpose the stopcock B is connected with a reservoir of the gas, by means of the tube D. The pump exhausts this gas, and forces it into the vessel K, in which the liquid is contained. The artificial gaseous waters are made by means of analogous apparatus.

The principle of the condensing pump has many applications, such as in the small pump used by plumbers for testing and for clearing gas pipes, in ventilating mines, in supplying air to blast furnaces, and so forth.

200. **Uses of the air-pump.**—A great many experiments with the air-pump have been already described. Such are the mercurial rain (13), the

fall of bodies in vacuo (77), the bladder (147), the bursting of a bladder (153), the Magdeburg hemispheres (154), and the baroscope (184).

The fountain in vacuo (fig. 171) is an experiment made with the air-pump, and shows the elastic force of the air. It consists of a glass vessel, A, provided at the bottom with a stopcock, and a tubulure which projects into the interior. Having screwed this apparatus to the air-pump it is exhausted, and, the stopcock being closed, it is placed in a vessel of water, R. Opening then the stopcock, the atmospheric pressure upon the water in the vessel makes it jet through the tubulure into the interior of the vessel, as shown in the drawing.

Fig. 171.

Fig. 172.

Fig. 172 represents an experiment illustrating the effect of atmospheric pressure on the human body. A glass vessel, open at both ends, being placed on the plate of the machine, the upper end of the cylinder is closed by the hand, and a vacuum is made. The hand then becomes pressed by the weight of the atmosphere, and can only be taken away by a great effort. And as the elasticity of the fluids contained in the organs is not counterbalanced by the weight of the atmosphere, the palm of the hand swells, and blood tends to escape from the pores.

By means of the air-pump it may be shown that air, by reason of the oxygen it contains, is necessary for the support of combustion and of life. For if we place a lighted taper under the receiver, and begin to exhaust the air, the flame becomes weaker as rarefaction proceeds, and is finally extinguished. Similarly an animal faints and dies if a vacuum is formed in a receiver under which it is placed. Mammalia and birds soon die in vacuo. Fish and reptiles support the loss of air for a much longer time. Insects can live several days in vacuo.

Substances liable to ferment may be kept in vacuo for a long time without alteration, as they are not in contact with oxygen, which is necessary for fermentation. Food kept in hermetically-closed cases, from which the air had been exhausted, has been found as fresh after several years as on the first day.

201. **Hero's fountain.**—Hero's fountain, which derives its name from its inventor, Hero, who lived at Alexandria, 120 B.C., depends on the elasticity of the air. It consists of a brass dish, D (fig. 173), and of two glass globes, M and N. The dish communicates with the lower part of the globe N by a long tube, B; and another tube, A, connects the two globes. A third tube passes through the dish D to the lower part of the globe M. This tube having been taken out, the globe M is partially filled with water, the tube is then replaced, and water is poured into the dish. The water flows through the tube B into the lower globe, and expels the air, which is forced into the upper globe; the air, thus compressed, acts upon the water, and makes it jet out as represented in the figure. If it were not for the resistance of the atmosphere and friction, the liquid would rise to a height above the water in the dish equal to the difference of the level in the two globes.

Fig. 173.

Fig. 174.

202. **Intermittent fountain.**—The *intermittent fountain* depends partly on the elastic force of the air and partly on the atmospheric pressure. It consists of a stoppered glass globe (C, fig. 174), provided with two or three capillary tubulures, D. A glass tube open at both ends reaches at one end to the upper part of the globe C; the other end terminates just above a little aperture in the dish B, which supports the whole apparatus.

The water with which the globe C is nearly two-thirds filled, runs out by the tubes D, as shown in the figure; the internal pressure at D being equal to the atmospheric pressure, together with the weight of the column of water CD, while the external pressure at that point is only that of the atmosphere. These conditions prevail so long as the lower end of the glass tube is open; that is, so long as air can enter C and keep the air in C at the same density as the external air; but the apparatus is arranged so that the orifice in the dish B does not allow so much water to flow out as it receives from the tubes D, in consequence of which the level gradually rises in the dish, and closes the lower end of the glass tube. As the external air cannot now enter the

globe C, the air becomes rarefied in proportion as the flow continues, until the pressure of the column of water CD, together with the tension of the air contained in the globe, is equal to this external pressure at D; the flow consequently stops. But as water continues to flow out of the dish B, the tube D becomes open again, air enters, and the flow recommences, and so on, as long as there is water in the globe C.

203. **The syphon.**—The syphon is a bent tube open at both ends, and with unequal legs (fig. 175). It is used in transferring liquids in the following manner:—The syphon is filled with some liquid, and, the two ends being closed, the shorter leg is dipped in the liquid, as represented, in fig. 175; or the shorter leg having been dipped in the liquid, the air is exhausted by applying the mouth at B. A vacuum is thus produced, the liquid in C rises and fills the tube in consequence of the atmospheric pressure. It will then run out through the syphon as long as the shorter end dips in the liquid.

Fig. 175.

To explain this flow of water from the syphon, let us suppose it filled and the short leg immersed in the liquid. The pressure then acting on C, and tending to raise the liquid in the tube, is the atmospheric pressure minus the height of the column of liquid DC. In like manner, the pressure on the end of the tube, B, is the weight of the atmosphere less the pressure of the column of liquid AB. But as this latter column is longer than CD, the force acting at B is less than the force acting at C, and consequently a flow takes place proportional to the difference between these two forces. The flow will therefore be more rapid in proportion as the difference of level between the aperture B and the surface of the liquid in C is greater.

It follows from the theory of the syphon that it would not work in vacuo, nor if the height CD were greater than that of a column of liquid which counterbalances the atmospheric pressure.


Fig. 176.

204. **The intermittent syphon.**—In the *intermittent* syphon the flow is not continuous. It is arranged in a vessel, so that the shorter leg is near the bottom of the vessel, while the longer leg passes through it (fig. 176). Being fed by a constant supply of water, the level gradually rises both in the vessel and in the tube to the top of the syphon, which it fills, and water begins to flow out. But the apparatus is arranged so that the flow of the syphon is more rapid than that of the tube which supplies the vessel, and consequently the level sinks in the vessel until the shorter branch no longer dips in the liquid; the syphon is then empty, and the flow

ceases. But as the vessel is continually fed from the same source, the level again rises, and the same series of phenomena is reproduced.

The theory of the intermittent syphon explains the natural intermittent springs which are found in many countries, and of which there is an excellent example near Giggleswick in Yorkshire. Many of these springs furnish water for several days or months, and then, after stopping for a certain interval, again recommence. In others the flow stops and recommences several times in an hour.

These phenomena are explained by assuming that there are subterranean fountains, which are more or less slowly filled by springs, and which are then emptied by fissures so occurring in the ground as to form an intermittent syphon.

205. **Different kinds of pumps.**—*Pumps* are machines which serve to raise water either by suction, by pressure, or by both efforts combined; they are consequently divided into *suction or lift pumps*, *force pumps*, and *suction and forcing pumps*.

The various parts entering into the construction of a pump are the barrel, the piston, the valves, and the pipes. The *barrel* is a cylinder of metal or

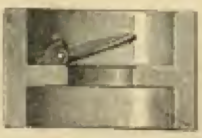

Fig. 177.

Fig. 178.

of wood, in which is the *piston*. The latter is a metal or wooden cylinder wrapped with tow, and working with gentle friction the whole length of the barrel.

The valves are discs of metal or leather, which alternately close the apertures which connect the barrel with the pipes. The most usual valves are the *clack valve* (fig. 177) and the *conical valve* (fig. 178). The first is a metal disc fixed to a hinge on the edge of the orifice to be closed. In order more effectually to close it, the lower part of the disc is covered with thick leather. Sometimes the valve consists merely of a leather disc, of larger diameter than the orifice, nailed on the edge of the orifice. Its flexibility enables it to act as a hinge.

The conical valve consists of a metal cone fitting in an aperture of the same shape. Below this is an iron loop, through which passes a bolt-head fixed to the valve. The object of this is to limit the play of the valve when it is raised by the water, and to prevent its removal.

206. **Suction pump.**—Fig. 179 represents a model of a suction pump such as is used in lectures, but which has the same arrangement as the pumps in common use. It consists, 1st, of a *glass cylinder*, B, at the bottom of which there is a valve, S, opening upwards; 2nd, of a *suction tube*, A, which dips into the reservoir from which water is to be raised; 3rd, of a *piston*, which is moved up and down by a rod worked by a handle, P. The piston is perforated by a hole; the upper aperture is closed by a valve, O, opening upwards.

When the piston rises from the bottom of the cylinder B, a vacuum is produced below, and the valve O is kept closed by the atmospheric pressure, while the air in the pipe A, in consequence of its elasticity, raises the valve S, and partially passes into the cylinder. The air being thus rarefied, water rises in the pipe until the pressure of the liquid column, together with the tension of the rarefied air which remains in the tube, counterbalances the pressure of the atmosphere on the water of the reservoir.

Fig. 179.

When the piston descends, the valve S closes by its own weight, and prevents the return of the air from the cylinder into the tube A. The air compressed by the piston opens the valve O, and escapes into the atmosphere by the pipe C. With a second stroke of the piston the same series of phenomena is produced, and after a few strokes the water reaches the cylinder. The effect is now somewhat modified: during the descent of the piston, the valve S closes, and the water raises the valve O, and passes above the piston by which it is lifted into the upper reservoir D. There is now no more air in the pump, and the water forced by the atmospheric pressure rises with the piston, provided that when it is at the summit of its course it is not more than 34 feet above the level of the water in which the tube A dips, for we have seen (156) that a column of water of this height is equal to the pressure of the atmosphere.

In practice the height of the tube A does not exceed 26 to 28 feet, for, although the atmospheric pressure can support a higher column, the vacuum produced in the barrel is not perfect, owing to the fact that the piston does not fit exactly on the bottom of the barrel. But when the water has passed the piston, it is the ascending force of the latter which raises it, and the height to which it can be brought depends on the force which moves the piston.

207. **Suction and force pump.**—The action of this pump, a model of which is represented in fig. 180, depends both on exhaustion and on pressure. At the base of the barrel, where it is connected with the tube A, there is a valve, S, which opens upwards. Another valve, O, opening in the same direction, closes the aperture of a conduit, which passes from a hole, *o*, near the valve S into a vessel M, which is called the *air chamber*. From this chamber there is another tube, D, up which the water is forced.

At each ascent of the piston B, which is solid, the water rises through the tube A into the barrel. When the piston sinks, the valve S closes, and the water is forced through the valve O into the reservoir M, and from thence into the tube D. The height to which it can be raised in this tube depends solely on the motive force which works the pump.

If the tube D were a prolongation of the tube *Jao*, the flow would be intermittent; it would take place when the piston descended, and would cease as soon as it ascended. But between these tubes there is an interval, which, by means of the air in the reservoir M, ensures a continuous flow. The water forced into the reservoir M divides into two parts, one of which, rising in D, presses on the water in the reservoir by its weight; while the other, in virtue of this pressure, rises in the reservoir above the lower orifice of the

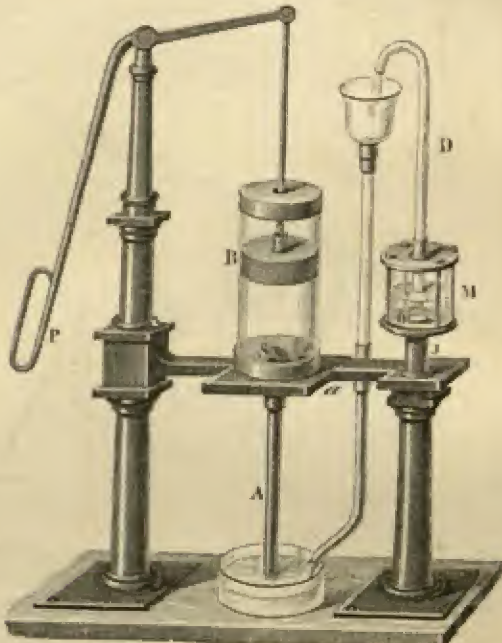

Fig. 180.

tube D, compressing the air above. Consequently, when the piston ascends, and no longer forces the water into M, the air of the reservoir, by the pressure it has received, reacts on the liquid, and raises it in the tube D, until the piston again descends, so that the jet is continuous.

208. **Load which the piston supports.**—In the suction pump, when once the water fills the pipe, and the barrel, as far as the spout, *the effort necessary to raise the piston is equal to the weight of a column of water, the base of which is this piston, and the height the vertical distance of the spout*

from the level of the water in the reservoir; that is, the height to which the water is raised. For if H is the atmospheric pressure, h the height of the water above the piston, and h' the height of the column which fills the suction tube A (fig. 180), and the lower part of the barrel, the pressure above the piston is obviously $H+h$, and that below is $H-h'$, since the weight of the column h' tends to counterbalance the atmospheric pressure. But as the pressure $H-h'$ tends to raise the piston, the effective resistance is equal to the excess of $H+h$ over $H-h'$, that is to say, to $h+h'$.

In the suction and force pump it is readily seen that the pressure which the piston supports is also equal to the weight of a column of water, the base of which is the section of the piston, and the height that to which the water is raised.

209. **Fire engine.**—The fire engine is a force pump in which a steady jet is obtained by the aid of an air chamber, and also by two pumps working alternately (fig. 181). The two pumps m and n, worked by the same lever

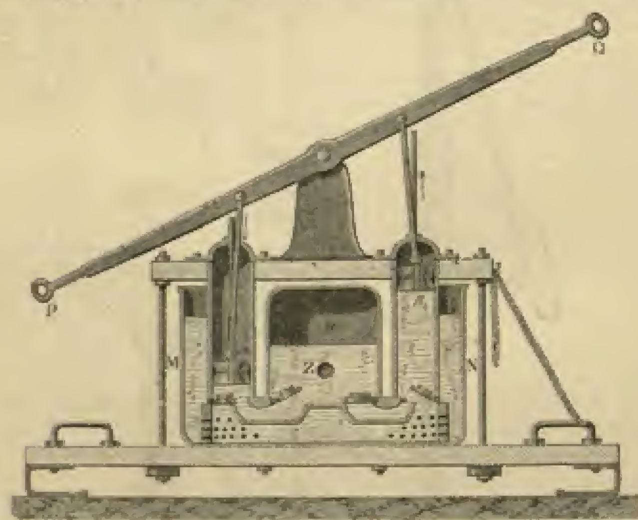

Fig. 181.

PQ, are immersed in a tank, which is kept filled with water as long as the pump works. From the arrangement of the valves it will be seen, that when one pump n draws water from the tank, the other m forces it into the *air chamber* R; whence, by an orifice Z, it passes into the delivery tube, by which it can be sent in any direction.

Without the air chamber the jet would be intermittent. But as the velocity of the water on entering the reservoir is less than on emerging, the level of the water rises above the orifice Z, compressing the air which fills the reservoir. Hence, whenever the piston stops, the air thus compressed, reacting on the liquid, forces it out during its momentary stoppage, and thus keeps up a constant flow.

210. **Velocity of efflux. Torricelli's theorem.**—Let us imagine an aperture made in the bottom of any vessel, and consider the case of a particle of liquid on the surface, without reference to those which are beneath. If this particle fell freely, it would have a velocity on reaching the orifice equal to that of any other body falling through the distance between the level of the liquid and the orifice. This, from the laws of falling bodies, is $\sqrt{2gh}$, in which g is the accelerating force of gravity, and h the height. If the liquid be maintained at the same level, for instance, by a stream of water running into the vessel sufficient to replace what has escaped, the particles will follow one another with the same velocity, and will issue in the form of a stream. Since pressure is transmitted equally in all directions, a liquid would issue from an orifice in the side with the same velocity provided the depth were the same.

The law of the velocity of efflux was discovered by Torricelli. It may be enunciated as follows:—*The velocity of efflux is the velocity which a freely falling body would have on reaching the orifice after having started from a state of rest at the surface.* It is algebraically expressed by the formula $v = \sqrt{2gh}$.

It follows directly from this law that the velocity of efflux depends on the depth of the orifice below the surface, and not on the nature of the liquid. Through orifices of equal size and of the same depth, water and mercury would issue with the same velocity, for although the density of the latter liquid is greater, the weight of the column, and consequently the pressure, is greater too. It follows further that the velocities of efflux are directly proportional to the square roots of the depth of the orifices. Water would issue from an orifice 100 inches below the surface with ten times the velocity with which it would issue from one an inch below the surface.

The quantities of water which issue from orifices of different areas are very nearly proportional to the size of the orifice, provided the level remains constant.

211. **Direction of the jet from lateral orifices.**—From the principle of the equal transmission of pressure, water issues from an orifice in the side of a vessel with the same velocity as from an aperture in the bottom of a vessel at the same depth. Each particle of a jet issuing from the side of a vessel begins to move horizontally with the velocity above mentioned, but it is at once drawn downward by the force of gravity in the same manner as a bullet, fired from a gun, with its axis horizontal. It is well known that the bullet describes a parabola (50) with a vertical axis, the vertex being the muzzle of the gun. Now since each particle of the jet moves in the same curve, the jet itself takes the parabolic form, as shown in fig. 182.

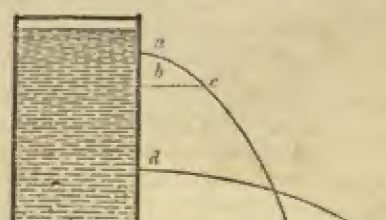

Fig. 182.

In every parabola there is a certain point called the *focus*, and the distance from the vertex to the focus fixes the magnitude of a parabola in

much the same manner as the distance from the centre to the circumference fixes the magnitude of a circle. Now it can easily be proved that the focus is as much below, as the surface of the water is above, the orifice. Accordingly the jets formed by water coming from orifices at different depths below the surface take different forms as shown in fig. 182.

212. **Height of the jet.**—If a jet issuing from an orifice in a vertical direction has the same velocity as a body would have which fell from the surface of the liquid to that orifice, the jet ought to rise to the level of the liquid. I does not, however, reach this; for the particles which fall hinder it. But by inclining the jet at a small angle with the vertical, it reaches about $\frac{8}{10}$ of the theoretical height, the difference being due to friction and to the resistance of the air. By experiments of this nature the truth of Torricelli's law has been demonstrated.

213. **Quantity of efflux. Vena contracta.**—If we suppose the sides of a vessel containing water to be thin, and the orifice to be a small circle whose area is A, we might think that the quantity of water E discharged in a second would be given by the expression $A\sqrt{2gh}$, since each particle has, on the average, a velocity equal to $\sqrt{2gh}$, and particles issue from each point of the orifice. But this is by no means the case. This may be explained by reference to fig. 179, in which AB represents an orifice in the bottom of a vessel—what is true in this case being equally true of an orifice in the side of the vessel. Every particle above AB endeavours to pass out of the vessel, and in so doing exerts a pressure on those near it. Those that issue near A and B exert pressures in the directions MM and NN; those near the centre of the orifice in the direction RQ, those in the intermediate parts in the directions PQ, PQ. In consequence, the water within the space PQP is unable to escape, and that which does escape, instead of assuming a cylindrical form, at first contracts, and takes the form of a truncated cone. It is found that the escaping jet continues to contract, until at a distance from the orifice about equal to the diameter of the orifice. This part of the jet is called the *vena contracta*. It is found that the area of its smallest section is about $\frac{5}{8}$ or 0·62 of that of the orifice. Accordingly, the true value of the efflux per second is given approximately by the formula

Fig. 183.

$$E = 0{\cdot}62A\sqrt{2gh}$$

or the actual value of E is about 0·62 of its *theoretical amount*.

214. **Influence of tubes on the quantity of efflux.**—The result given in the last article has reference to an aperture in a thin wall. If a cylindrical or conical efflux tube or *ajutage* is fitted to the aperture, the amount of the efflux is considerably increased, and in some cases falls but a little short of its theoretical amount.

A short cylindrical ajutage, whose length is from two to three times its diameter, has been found to increase the efflux per second to about $0{\cdot}82A\sqrt{2gh}$. In this case, the water on entering the ajutage forms a contracted vein (fig. 184), just as it would do on issuing freely into the air; but afterwards it expands, and, in consequence of the adhesion of the water to the interior surface of the tube, has, on leaving the ajutage, a section

greater than that of the contracted vein. The contraction of the jet within the ajutage causes a partial vacuum. If an aperture is made in the ajutage, near the point of greatest contraction, and is fitted with a vertical tube, the other end of which dips into water (fig. 184,) it is found that water rises in the vertical tube, thereby proving the formation of a partial vacuum.


Fig. 184.

If the ajutage has the form of a conic frustrum whose larger end is at the aperture, the efflux in a second may be raised to $0·92A\sqrt{2gh}$, provided the dimensions are properly chosen. If the smaller end of a frustrum of a cone of suitable dimensions be fitted to the orifice, the efflux may be still further increased, and fall very little short of the theoretical amount.

When the ajutage has more than a certain length, a considerable diminution takes place in the amount of the efflux: for example, if its length is 48 times its diameter, the efflux is reduced to $0·63A\sqrt{2gh}$. This arises from the fact, that, when water passes along cylindrical tubes, the resistance increases with the length of the tube; for a thin layer of liquid is attracted to the walls by adhesion, and the internal flowing liquid rubs against this. The resistance which gives rise to this result is called *hydraulic friction*: it is independent of the material of the tube, provided it be not roughened; but depends in a considerable degree on the viscosity of the liquid; for instance, ice-cold water experiences a greater resistance than lukewarm water.

According to Prony, the mean velocity v of water in a cast-iron pipe, ot the length l, and the diameter d, under the pressure p, is in metres

$$v = 26·8\sqrt{\frac{dp}{l}}.$$

By means of hydraulic pressure Tresca has submitted solids such as silver, lead, iron and steel, powders like sand, soft plastic substances such as clay, and brittle bodies like ice, to such enormous pressures as 100,000 kilogrammes, and has found that they then behave like fluid bodies. His experiments show also that these bodies transmit pressure equally in all directions, when this pressure is considerable enough.

215. **Efflux through capillary tubes.**—This was investigated by Poisseuille by means of the apparatus represented in fig. 185, in which the capillary tube AB is sealed to a glass tube on which a bulb is blown. The volume of the space between the marks M and N is accurately determined, and the apparatus having been filled with the liquid under examination by suction, the apparatus is connected at the end M, with a reservoir of compressed air, in which the pressure is measured by means of a mercury manometer. The time is then noted which is required for the level of the liquid to sink from M to N, the pressure remaining constant. Poisseuille thus found that q, the quantity which flows out in a given time, is represented by the formula

$$q = kp\frac{d^4}{l}$$

where p is the pressure, d the diameter, and l the length of the tube, while k is a constant, which varies with the nature of the liquid; and is greatly influenced by the temperature. An increase from 0° to 60° C increases the quantity threefold.

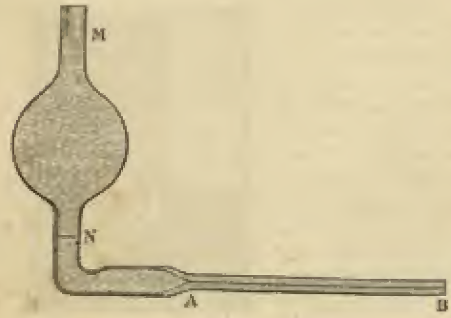

Fig. 185.

216. **Form of the jet.**—After the contracted vein, the jet has the form of a solid rod for a short distance, but then begins to separate into drops, which present a peculiar appearance. They seem to form a series of ventral and nodal segments (fig. 186). The ventral segments consist of drops extended in a horizontal direction, and the nodal segments in a longitudinal direction.

And as the ventral and nodal segments have respectively a fixed position, each drop must alternately become elongated and flattened while it is falling (fig. 187). Between any two drops there are smaller ones, so that the whole jet has a tube-like appearance.

If the jet is momentarily illuminated by the electric spark its structure is well seen; the drops appear then to be stationary, and separate from each other.

If the aperture is not circular the form of the jet undergoes curious changes.

217. **Hydraulic tourniquet.**—If water be contained in a vessel, and an aperture be made in one of the sides, the pressure at this point is removed, for it is expended in sending out the water: but it remains on the other side; and if the vessel were movable in a horizontal direction, it would move in a direction opposite that of the issuing jet. This is illustrated by the apparatus known as the *hydraulic tourniquet* or *Barker's mill* (fig. 188). It consists of a glass vessel, M, containing water, and capable of moving about its vertical axis. At the lower part there is a tube, C, bent horizontally in opposite directions at the two ends. If the vessel were full of water and the tubes closed, the pressure on the sides of C would balance each other, being equal and acting in contrary directions; but, being open, the water runs out, the pressure is not exerted on the open part, but only on the opposite side, as shown in the figure A. And this pressure, not being neutralised by an opposite pressure, imparts a rotatory motion in the direction of the arrow, the velocity of which increases with the height of the liquid and the size of the aperture.

The same principle may be illustrated by the following experiment. A tall cylinder containing water and provided with a lateral stopcock near the bottom is placed on a light shallow dish on water, so that it easily floats. On opening the stopcock so as to allow water to flow out, the vessel is observed to move in a direction diametrically opposite to that in which the

water is issuing. Similarly, if a vessel containing water be suspended by a string, on opening an aperture in one of the sides, the water will jet out, and the vessel be deflected away from the vertical in the opposite direction.

Segner's water-wheel and the reaction machine depend on this principle. So also do rotating fire-works; that is, an unbalanced reaction from the heated gases which issue from openings in them, gives them motion in the opposite direction.

218. **Water-wheels. Turbines.**—When water is continuously flowing from a higher to a lower level, it may be used as a motive power. The motive power of water is utilised by means of *water-wheels*; that is, by wheels provided with buckets or float-boards at the circumference, and on which the water acts either by pressure or by impact.

Fig. 186.

Fig. 187.

Fig. 188.

Water-wheels turn in a vertical plane round a horizontal axis, and are of two principal kinds, *undershot* and *overshot*.

In *undershot* wheels the float-boards are at right angles to the circumference of the wheel. The lowest float-boards are immersed in the water which flows with a velocity depending on the height of the fall. Such wheels are applicable where the quantity of water is great, but the fall inconsiderable. *Overshot* wheels are used with a small quantity of water which has a high fall, as with small mountain streams. On the circumference of the wheel there are buckets of a peculiar shape. The water falls into the buckets on the upper part of the wheel, which is thus moved by the weight of the water, and as each bucket arrives at the lowest point of revolution it discharges all the water, and ascends empty.

The *turbine* is a horizontal water-wheel, and is similar in principle to the

hydraulic tourniquet (217). But instead of the horizontal tubes there is a horizontal drum, containing curved vertical walls; the water, in issuing from the turbine, pressing against these walls, exerts a reaction, and turns the whole wheel about a vertical axis. Turbines have the advantage of being of small bulk for their power, and equally efficient for the highest and the lowest falls.

In places in which a high-pressure water supply is available, a form of *water motor* has of late come into use. The water is led from pipes into a cylinder, in which is a piston. By means of a special arrangement called the *distributor*, which will be more fully described under the steam engine, the water is alternately led above and below the piston, and therefore alternately presses it up and down. This motion of the piston is transmitted by suitable mechanical contrivances to the rest of the machine.

Instruments of this kind are made which, with a pressure of two atmospheres and a cylinder whose diameter is 4 c.m., give about $\frac{1}{8}$ of a horse power with a consumption of about 530 gallons of water in an hour.

Water-power is usually represented by the weight of the water multiplied into the height of the available fall; or it may also be represented by half the product of the mass into the square of the velocity. Both measurements give the same result (61).

The water power of the Niagara Falls is calculated to be equal to four and a half millions of horse-power.

The total theoretical effect of a water-power is never realised; for the water, after acting on the wheel, still retains some velocity, and therefore does not impart the whole of its velocity to the wheel; in many cases water flows past without acting at all; if the water acts by impact, vibrations are produced which are transmitted to the earth and lost; the same effect is produced by the friction of water over an edge of the sluice, in the channel which conveys it, or against the wheel itself, as well as by the friction of this latter against the axle. A wheel working freely in a stream, as with the corn mills on the Rhine near Mainz, does not utilise more than 20 per cent. of the theoretical effect, while one of the more perfect forms of turbines will work up to over 80 per cent. Water engines in this respect exceed steam engines, which on the average do not use more than 10 per cent. of the power represented by the coal they burn.

Fig. 189.

219. **Mariotte's bottle, its use.**—Mariotte's bottle presents many curious effects of the pressure of the atmosphere, and furnishes a means of obtaining a constant flow of water. It consists of a large narrow-mouthed bottle in the neck of which there is a tightly-fitting cork (fig. 189). Through this a tube passes open at both ends. In the sides of the bottle there are three tubulures, each with a narrow orifice, and which can be closed at will.

The bottle and the tube being quite filled with water, let us consider what will be the effect of opening successively one of the tubulures, *a*, *b*, and *c*, supposing, as represented in the figure, that the lower extremity of *g* is between the tubulures *b* and *c*.

i. If the tubulure b is open the water flows out, and the surface sinks in the tube g until it is on the same level as b when the flow stops. This flow arises from the excess of pressure at the point e over that at b. The pressure at e is the same as the pressure of the atmosphere. But when once the level is the same at b and at e, the efflux ceases, for the atmospheric pressure on all points of the same horizontal layer, be, is the same (100).

ii. If now the tubulure b is closed, and a opened, no efflux takes place; on the contrary, air enters by the orifice a, and water ascends in the tube g, as high as the layer ad, and then equilibrium is established.

iii. If the orifices a and b are closed, and c opened, an efflux having constant velocity takes place, as long as the level of the water is not below the open end, l, of the tube. Air enters bubble by bubble at l, and takes the place of the water which has flowed out.

In order to show that the efflux at the orifice c is constant, it is necessary to demonstrate that the pressure on the horizontal layer ch is always equal to that of the atmosphere in addition to the pressure of the column hl. Now suppose that the level of the water has sunk to the layer ad. The air which has penetrated into the flask supports a pressure equal to that of the atmosphere diminished by that of the column of liquid pn, or $H - pn$. In virtue of its elasticity this pressure is transmitted to the layer ch. But this layer further supports the weight of a column of water, pm, so that the pressure at m is really $pm + H - pn$, or $H + mn$, that is to say, $H + hl$.

In the same manner it may be shown that this pressure is the same when the level sinks to b, and so on as long as the level is higher than the aperture l. The pressure on the layer ch is therefore constant, and consequently the velocity of the efflux. But when once the level is below the point l, the pressure decreases, and with it the velocity.

To obtain a constant flow by means of Mariotte's bottle, it is filled with water, and the orifice which is below the tube l is opened. The rapidity of the flow is proportional to the square root of the height hl.

Now, if σ be the specific gravity of the gas as compared with air, which is $\frac{1}{773 \cdot 3}$ lighter than water, $\rho \times 773 \cdot 3 = \sigma$, or $\rho = \frac{\sigma}{773 \cdot 3}$,

$$u^2 = \frac{3 \times 13 \cdot 596 \times 0 \cdot 76 \times 9 \cdot 8115 \times 773 \cdot 3}{\sigma}$$

which gives $u = \frac{485^m}{\sqrt{\sigma}}$; that is, that for atmospheric air the mean velocity of the particles is 485 metres in a second. For other gases we have, expressed in the same units,

$$\begin{aligned} O &= 461 \\ N &= 492 \\ H &= 1844. \end{aligned}$$

In a gas the velocities of the particles are unequal; for, even supposing that they were all originally the same, it is not difficult to see that they would soon alter. For imagine a particle to be moving parallel to one side, and to be struck centrically by another moving at right angles to the direction of its motion, the particle struck would proceed on its new path with increased velocity, while the striking particle would rebound in a different direction with a smaller velocity.

Notwithstanding the accidental character of the velocity of any individual particle in such a mass of gas as we have been considering, there will, at any one given time, be a certain average distribution of velocities. Now, from considerations based on the theory of probabilities, it follows that some velocities will be more probable than others—that there will, indeed, be one velocity which is more probable than any other. This is called the *most probable* velocity. The *mean velocity* of the particle, as found above, is not this, nor is it the same as the arithmetical mean of all the velocities; it may be defined to be that velocity which, if all the molecules possessed it, the mean energy of the molecular impacts against the side would be the same as that which actually exists. This mean velocity is about $\frac{1}{12}$ greater than the arithmetical mean velocity, and is $1\frac{1}{4}$ that of the most probable single velocity.

295. **General effects of heat.**—The general effects of heat upon bodies may be classed under three heads. One portion is expended in raising the temperature of the body; that is, in increasing the vis viva of its molecules. In the second place, the molecules of bodies have a certain attraction for each other, to which is due their relative position; hence a second portion of heat is consumed in augmenting the amplitude of the oscillations, by which an increase of volume is produced, or in completely altering the relative positions of the molecules, by which a change of state is effected. These two effects are classed as *internal work*. Thirdly, since bodies are surrounded by atmospheric air which exerts a certain pressure on their surface, this has to be overcome or lifted through a certain distance. The heat or work required for this is called the *external work*.

If Q units of heat are imparted to a body, and if A be the quantity of heat which is equivalent to the unit of work; then if W is the amount of heat which serves to increase the temperature, I that required to alter the